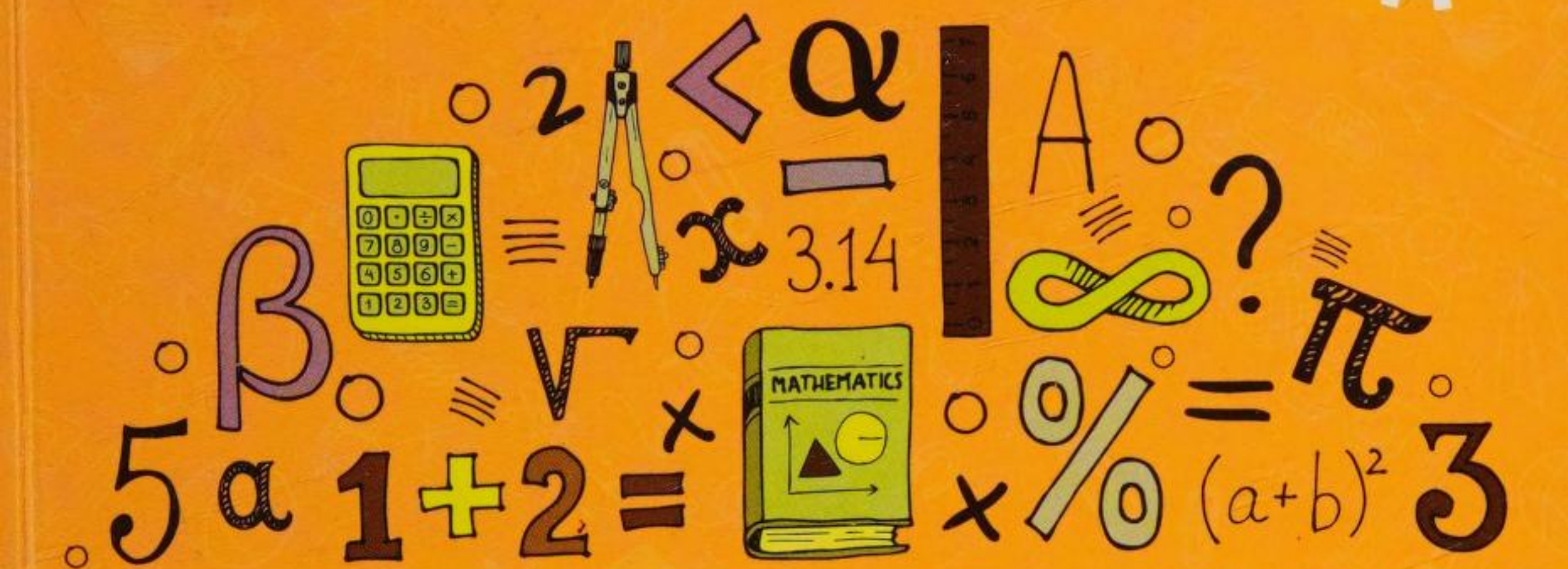
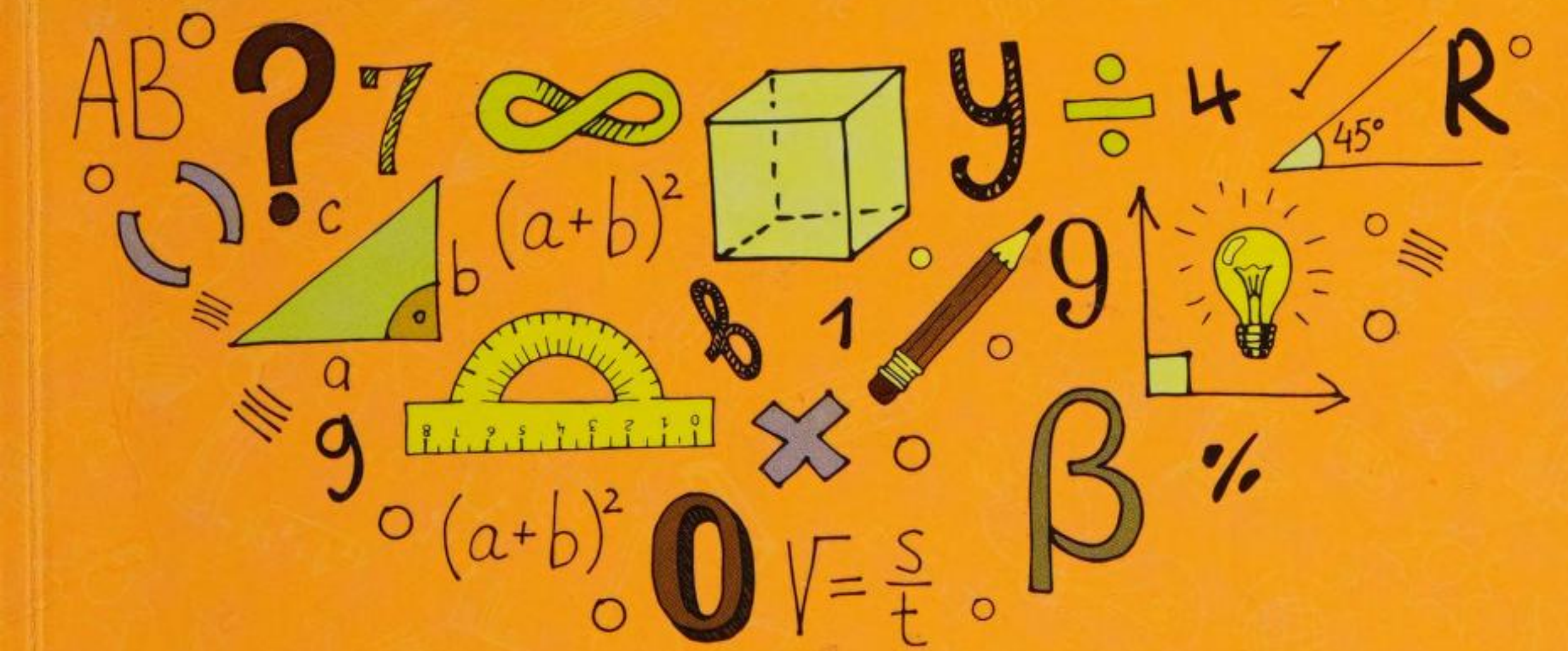
MATH-A-MAGIC 2



MATHS PUZZLES, GAMES AND TRICKS



RAJESH KUMAR THAKUR

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MATH-A-MAGIC 2:
Maths Puzzles, Games and Tricks

Dr Rajesh Kumar Thakur is a mathematics teacher by profession with 15 years of teaching experience. He has a master's degree in Mathematics, Operational Research, Education and Sociology and a doctorate in Education. He has received many awards—National Best Teacher Award, Special Achievement Award, Mathematical Accolade of Honour, Lalit Kishore Pandey Yuva Lelhan Award etc. He is a resource person for CBSE, SSUN, SCERT, AIRMC, VIPNET and other organisations that are working for the popularisation of mathematics. He is an honorary director and chairman of the award selection committee of All India Ramanujan Maths Club. He has written 50 books and over 300 articles for magazines, newspapers, blogs etc.

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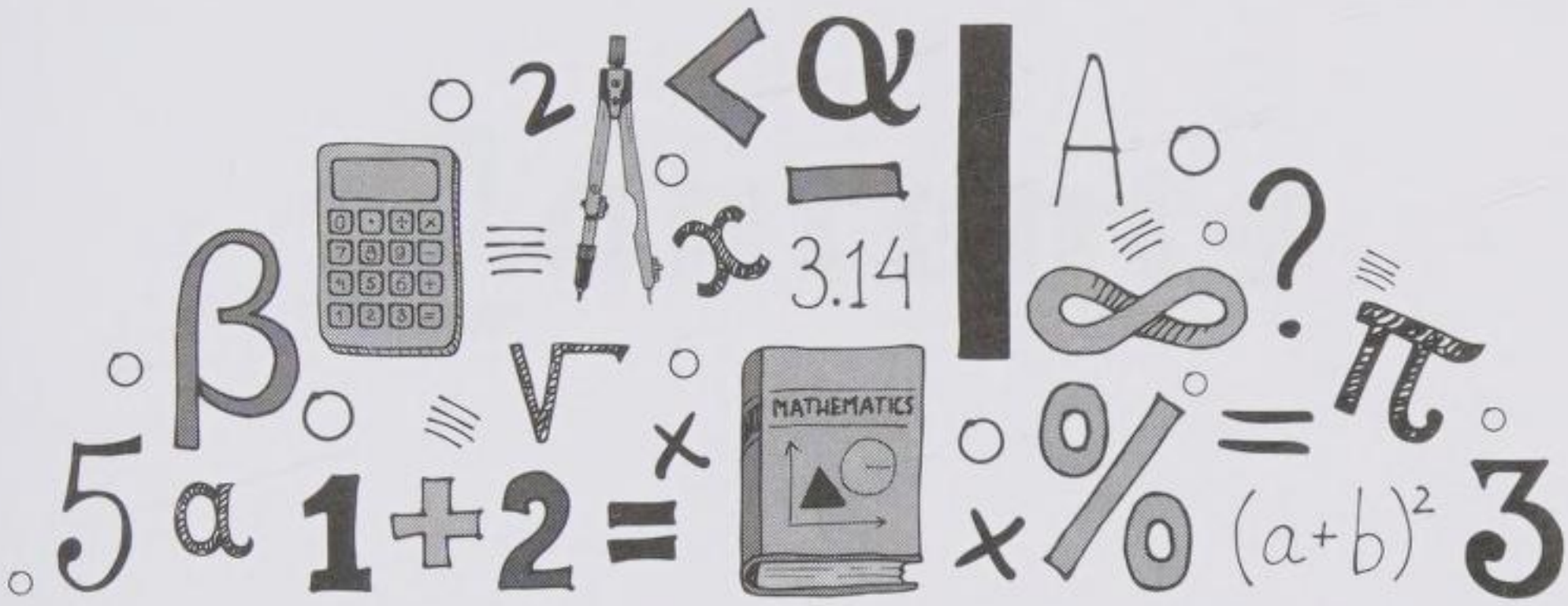
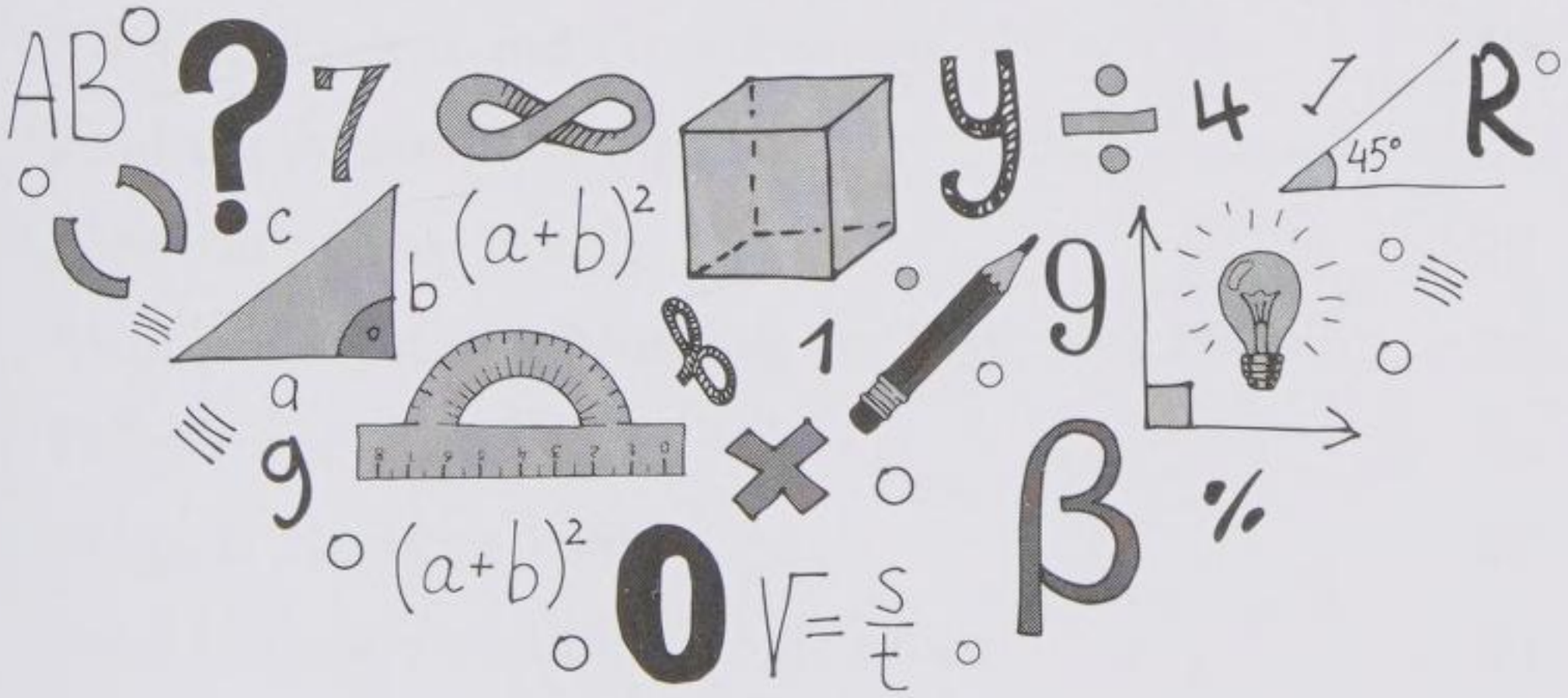


MATHS PUZZLES, GAMES AND TRICKS



RAJESH KUMAR THAKUR

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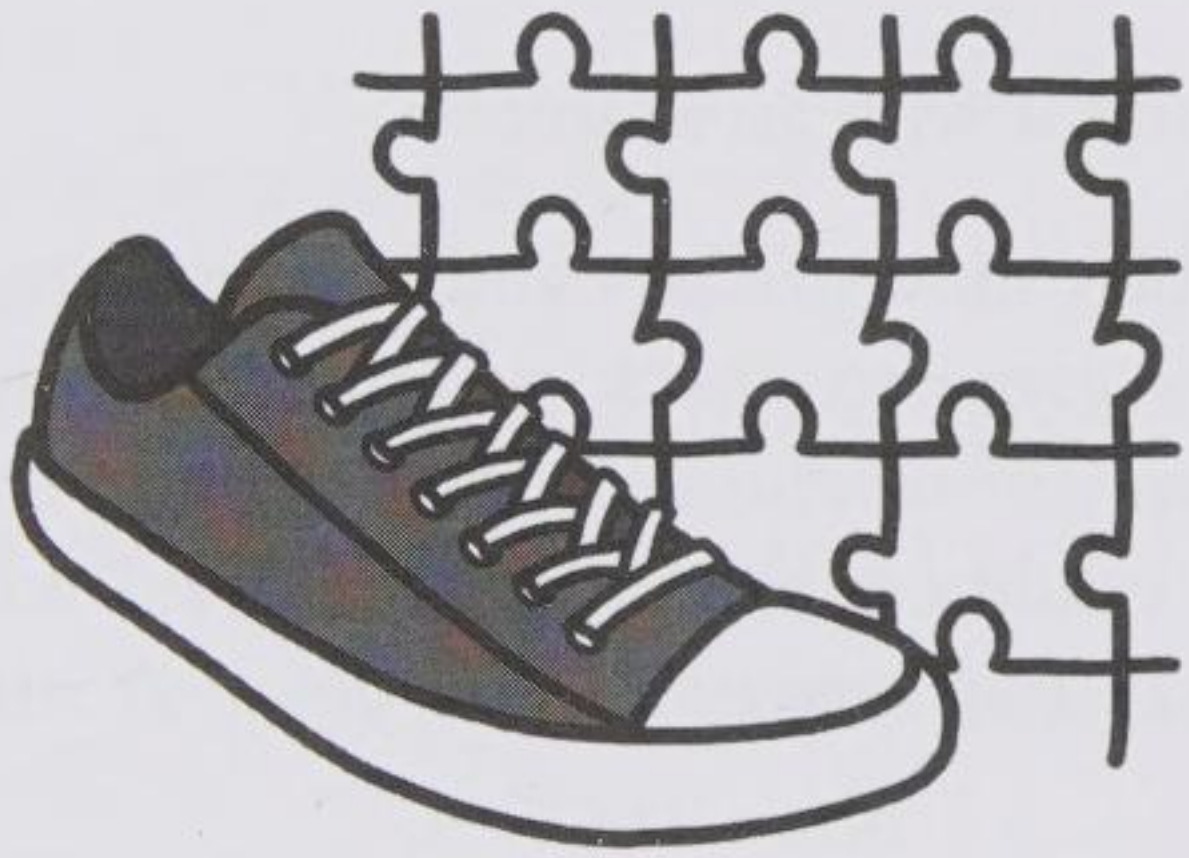
Shoe Magic



Mathematics is always considered a dreadful subject. Here is a puzzle that'll help make it interesting. This puzzle will tell you how to know your friend's age and his shoe size.

Let's learn a simple algebra trick to know the age and shoe size of your friends and entertain them.

Ask a friend to follow these simple instructions and tell you the final answer:-

- Take your shoe size. The shoe size should not be in halves.
- Multiply it by 5.

- Add 50 to the previous result.
- Multiply it by 20.
- Add 1017 (in the year 2017), 1018 (in 2018).
- Subtract the year you were born in from the previous result.

You will get a three-digit number at the end. The two digits in the ones' and tens' places will tell your friend's age while the number in the hundreds' place will be his/her shoe size.

Let's understand it with an example.

My son Nilabh's shoe size is 4 and he was born in 2010. Let's play this game to check whether the trick works or not. As you can see, he is 7 years old and since I am checking the validity of this puzzle in 2017, I will have to add 1017 in the step below.

Shoe size ⇨ 4

Multiply it by 5 ⇨ $4 \times 5 = 20$

Add 50 ⇨ $20 + 50 = 70$

Multiply the previous result by 20 ⇨ $20 \times 70 = 1400$

Add 1017 ⇨ $1400 + 1017 = 2417$

Subtract the birth year ⇨ $2417 - 2010 = 407$

Here the digit at hundred's place,—4—

will give you the shoe size

The remaining two digits ⇨ 07 = present age

Now the big question is: How did I know the answer?

Basically this puzzle is based on algebra and if you

understand algebra a little bit, you can crack the principle behind it.

➲ Do you know that algebra got its name from the Arabic word *al-jabr* meaning "re-union"? The first book of algebra *Ilm al-jabr wa'l-muḳābala* was written by Al-Khwarizmi.

Let's understand the puzzle algebraically.

Let your shoe size be x. Suppose you were born in 2010. Work out the following steps in your copy.

Shoe size = x

Multiply the shoe size by 5 = $5x$

Add 50 ⇨ $5x + 50$

Multiply it by 20 ⇨ $20\ (5x + 50) = 100x + 1000$

Add 1017 ⇨ $100x + 2017$

Subtract the year you were born in ⇨$100x + 2017 - 2010$
$= 100x + 07$

Since the shoe size is taken as x where $1 \leq x \leq 9$ then obviously you will get a three-digit number where the last two digits denote your age and the value of x will be the shoe size.

This puzzle will work even for shoe sizes 10 to 99, and in that case, the result at the end will be a four-digit number.

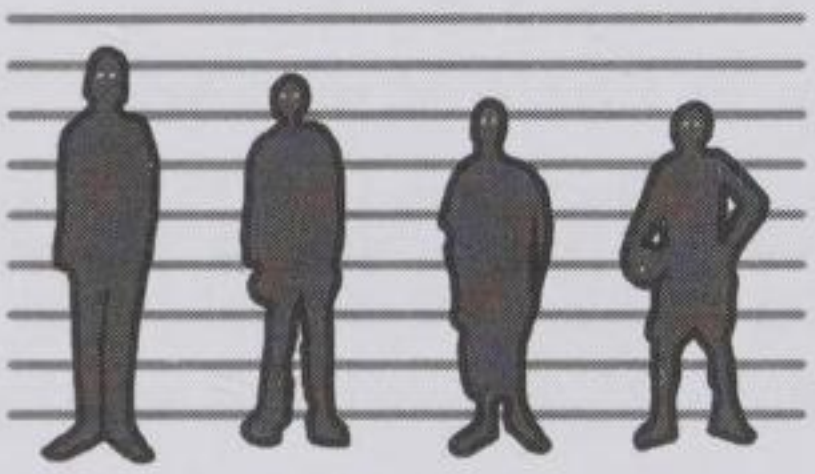
In case the shoe size is taken between 1 and 9, the result will be a three-digit number.

Hope you have understood the puzzle with valid proof.

So enjoy this puzzle and play it with your friends. Don't forget to tell them the algebraic proof behind it!

2

Age and Height of Your Friend



You can play this game and impress your friends with your mathematical wizardry. Call some of your friends and tell them that you have learnt a magic trick to guess the height and age of any person correctly. Give them a pen, paper and calculator each, if possible and direct them to follow these instructions and get ready to be astonished.

- Multiply your age by 2.
- Add 7 to it.
- Multiply the result by 5.
- Subtract 5 from it.
- Divide the result by 10.
- Multiply the last result by 1000.
- Add height (in centimetres).

Take some rest. You have done a splendid job by following the instructions well. Now, let me tell you the age and height of each of your friends in that order.

Take the final number from your friend and subtract 3000 from it and you will have a number left with you. The last three digits (Hundreds Tens Ones) will be the height in centimeters and the remaining two digits will be the age of your friend. Let me take one example to show you how it works.

Example:- Kanchan is 26 years old and 157 cms tall. Let's check the data with the method discussed above.

- Multiply your age by 2 ⇨ $26 \times 2 = 52$.
- Add 7 to it ⇨ $52 + 7 = 59$.
- Multiply the result by 5 ⇨ $59 \times 5 = 295$.
- Subtract 5 from it ⇨ $295 - 5 = 290$.
- Divide the result by 10 ⇨ $290 \div 10 = 29$.
- Multiply the last result by 1000 ⇨ $29 \times 1000 = 29000$.
- Add height in centimeters ⇨ $29000 + 157 = 29157$.

Final result = 29157

Subtract 3000 ⇨ 29157 – 3000 = 26157

Here, height = 157 = Last three digits

Age = 26 = Remaining two digits

Now the big question is: How did we get to the answer?

Here the mathemagician will not inquire anything beforehand; on the contrary, the person who is playing the game will only give the final result obtained at the end. The maths wizard's

work is to give the other person instructions and he will do the calculation on his part. The basis of such puzzles is to let the other person do the calculation, and his/her result will be revealing what he/she has used beforehand. This is not magic: there's a mathematical approach to understand the logic. So let me explain it to you.

If x = your age and y = your height, then follow the instructions and check what the results are in the end.

- Multiply your age by 2 ⇨ $2x$.
- Add 7 to it ⇨ $2x + 7$.
- Multiply the result by 5 ⇨ $5\ (2x + 7) = 10x + 35$.
- Subtract 5 from it ⇨ $10x + 35 - 5 = 10x + 30$.
- Divide the result by 10 ⇨ $x + 3$.
- Multiply the last result by 1000 ⇨ $1000x + 3000$.
- Add height in centimeters ⇨ $1000x + 3000 + y$.

If we subtract 3000 from the above result, we will be left with $1000x + y$ which clearly indicates that the last three digits will be the height and the rest will be the age of the person.

The person whose height is to be guessed doesn't know why we have subtracted 3000 at the end. Subtracting 3000 from the final result will give us a five-digit number that will help us guess the correct answer.

The algebraic equation explained here is to understand the logic behind such a game and the person who is guessing is not going to reveal the algebraic explanation.

The objective is to make mathematics a friendly subject with the help of calculations and a little algebra.

3

Money Spent According to the Date

Rahul is in the habit of spending money according to the date of the month. In simple words, he spends ₹21 on the 21st and ₹15 on the 15th. Last Sunday, his father gave him ₹100 to spend in the coming week. He calculated a total expenditure of 5 consecutive days—Monday to Friday—and he found that he had ₹37 left i.e. he had spent ₹63 in 5 days.

Can you guess the dates?

Solution:- Try to form an equation, with x being the first day, and increase 1 rupee daily because he spends the amount in 5 consecutive days. The dates will be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$.

According to the condition laid in the puzzle,

$x + x + 1 + x + 2 + x + 3 + x + 4 = 63$

$5x + 10 = 63$

If we put $x = 11$ then we will get $65 > 63$ and for $x = 10$, we get $60 < 63$.

This shows that the dates can't be in increasing order and a few dates may be less than 10 (referring to the tenth day of the month) and some others greater than 10. This can happen only in a case when the dates fall over two months as the last few days of one month and initial few days of the next month. It means we have to select the date arbitrarily. If we begin with date 30 and consider it a 30-day month, then the next four days—1, 2, 3, 4—will not add up to 63. This clearly indicates that the last two days should give the highest total, but we can't take a month with 31 days so we are now left with an option of a month with 28 or 29 days.

If we take February 28, and 1st, 2nd, 3rd and 4th of March, this again fails to satisfy the total sum. Let's then go for the month of February with 29 days and begin with date 28. Then the five days will be 28, 29 and three days of March, namely 1, 2 and 3.

Sum total of 5 days ⇨ $28 + 29 + 1 + 2 + 3 = 63$.

Hurray!

We have reached the solution. The dates on which Rahul spent his money are obviously February 28, February 29, March 1, March 2 and March 3.

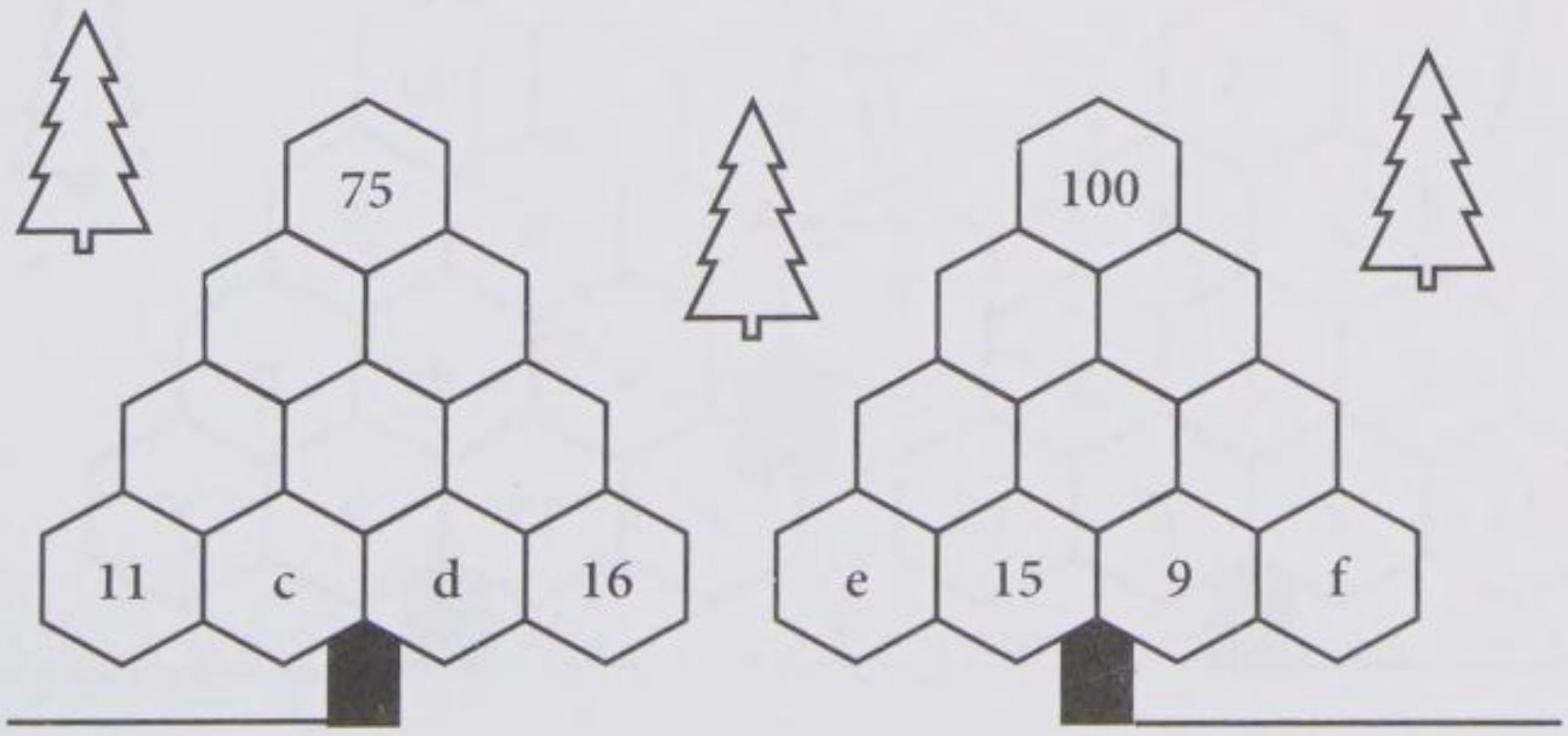
4

Magic Tree

Have you ever decorated a Christmas tree?

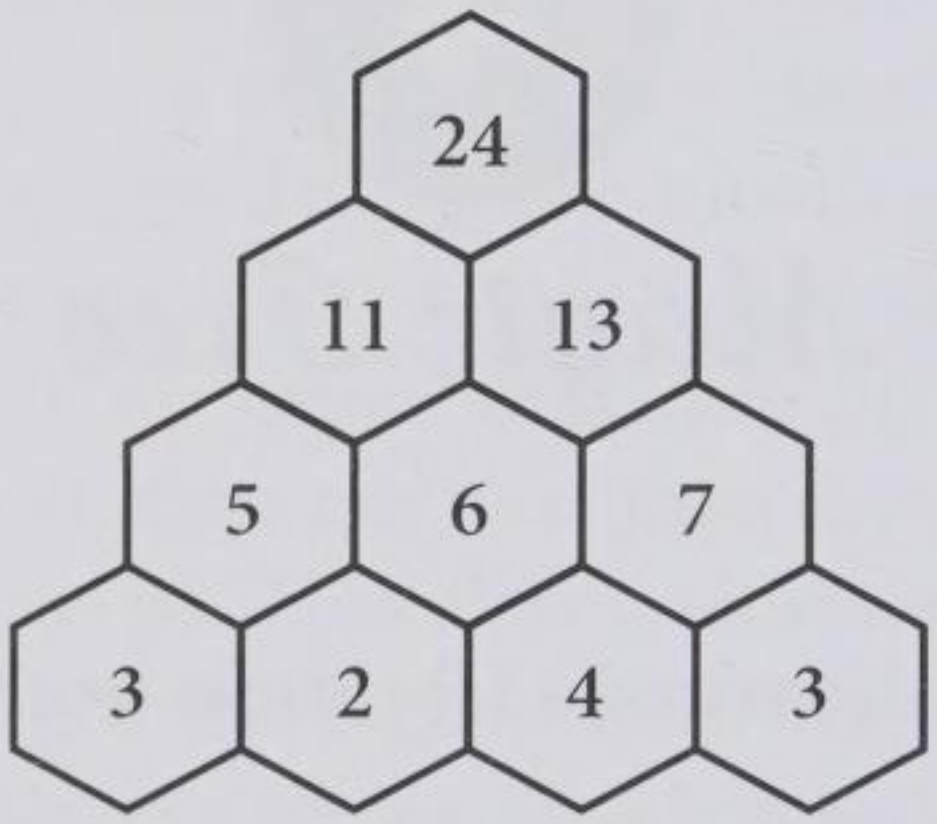
Nilabh and Nishtha were busy preparing a Number Christmas tree—a project given by their class teacher. They tried their best to put the correct numbers in the empty boxes, but failed. They decided to take the help of their father who is a mathematics teacher.

Their father turned them down. Instead, he promised to take them to the nearby mall in the evening if either Nilabh or Nishtha got at least two solutions for each of the number trees in the next 30 minutes.



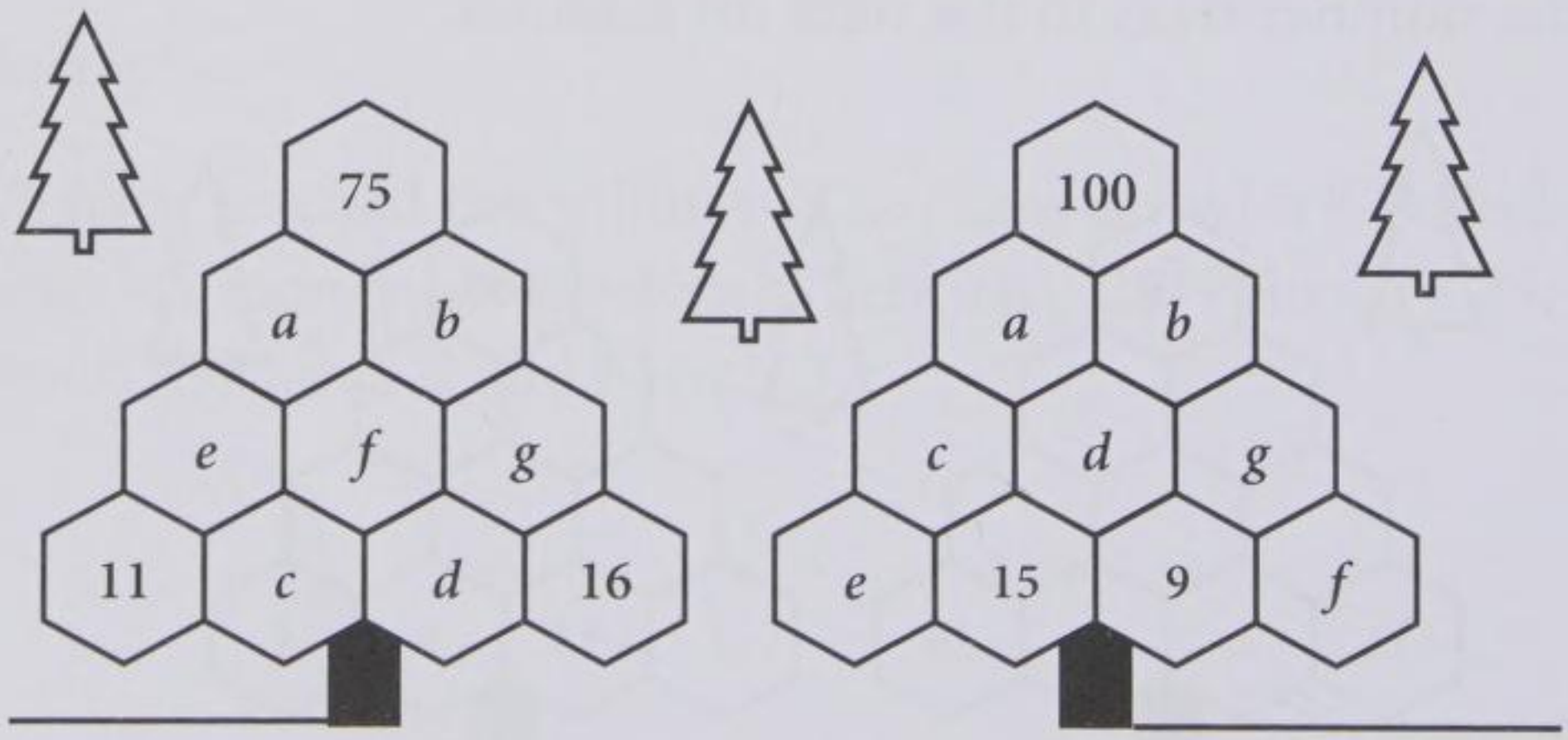
Nilabh: Papa, I am ready to solve this riddle, provided you give me a clue and an example and show me how to proceed.

Papa: Ok, let me give you an example.



In every row (except the bottom-most one), each number is the sum of the numbers in the row below it. For e.g. 24 at the top is the sum of the two numbers below it—11 and 13. There is no rule of placing the letters. One can place any alphabet in any order.

Nilabh then decided to accept the challenge and put the letters in the empty boxes.



Let's begin with the second tree.

$c = e + 15$

$d = 15 + 9 = 24$

$g = 9 + f$

$a = c + d = e + 15 + 24 = e + 39$

$b = d + g = 24 + 9 + f = 33 + f$

$e + f = 28$

Finally, $a + b = 100$

$$e + 39 + 33 + f = 100$$

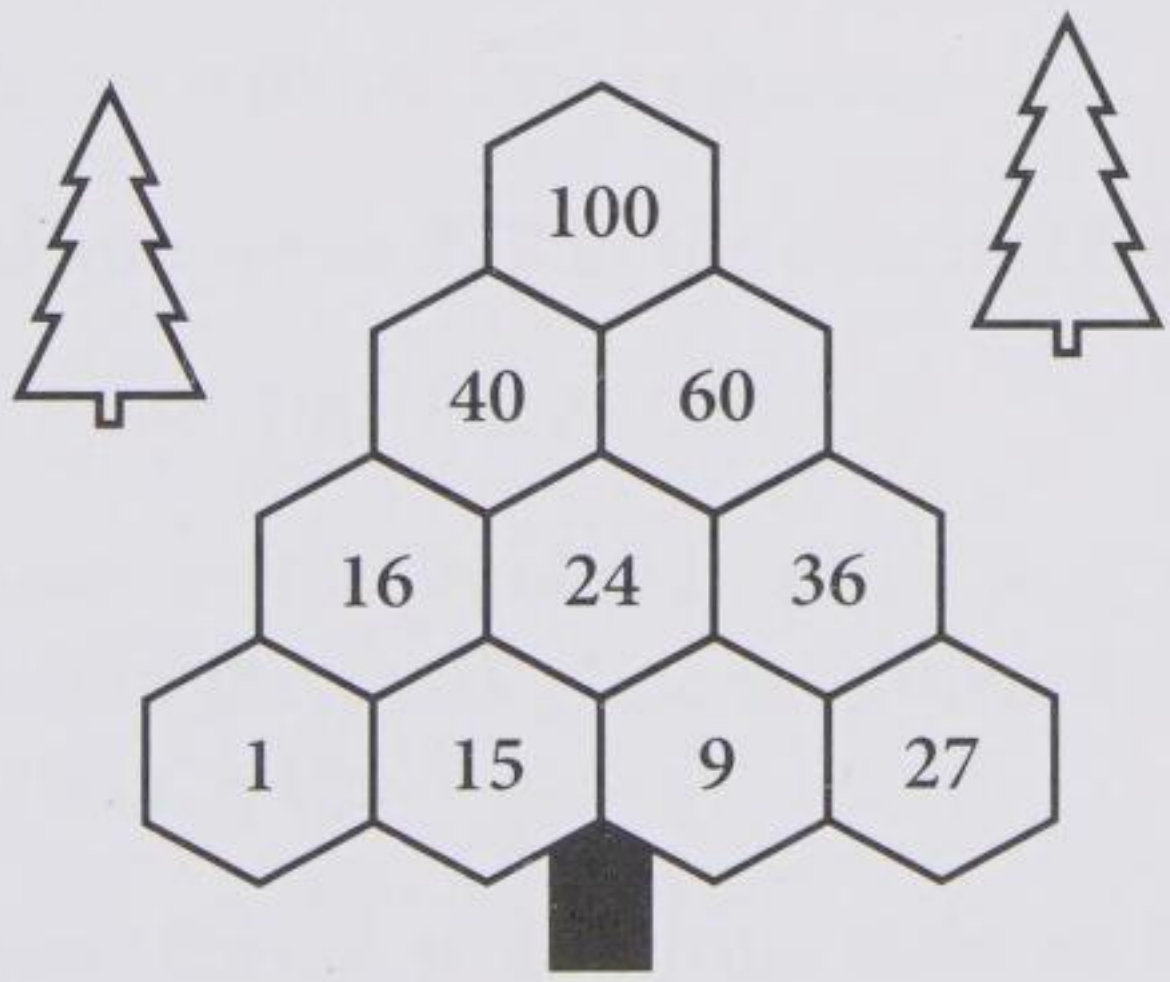
The whole tree will depend on the value of e and f. Let's consider the case:

If $e = 1$ then $f = 27$

$c = 1 + 15 = 16$ $\quad$ $g = 27 + 9 = 36$

$a = 1 + 39 = 40$ $\quad$ $b = 33 + 27 = 60$

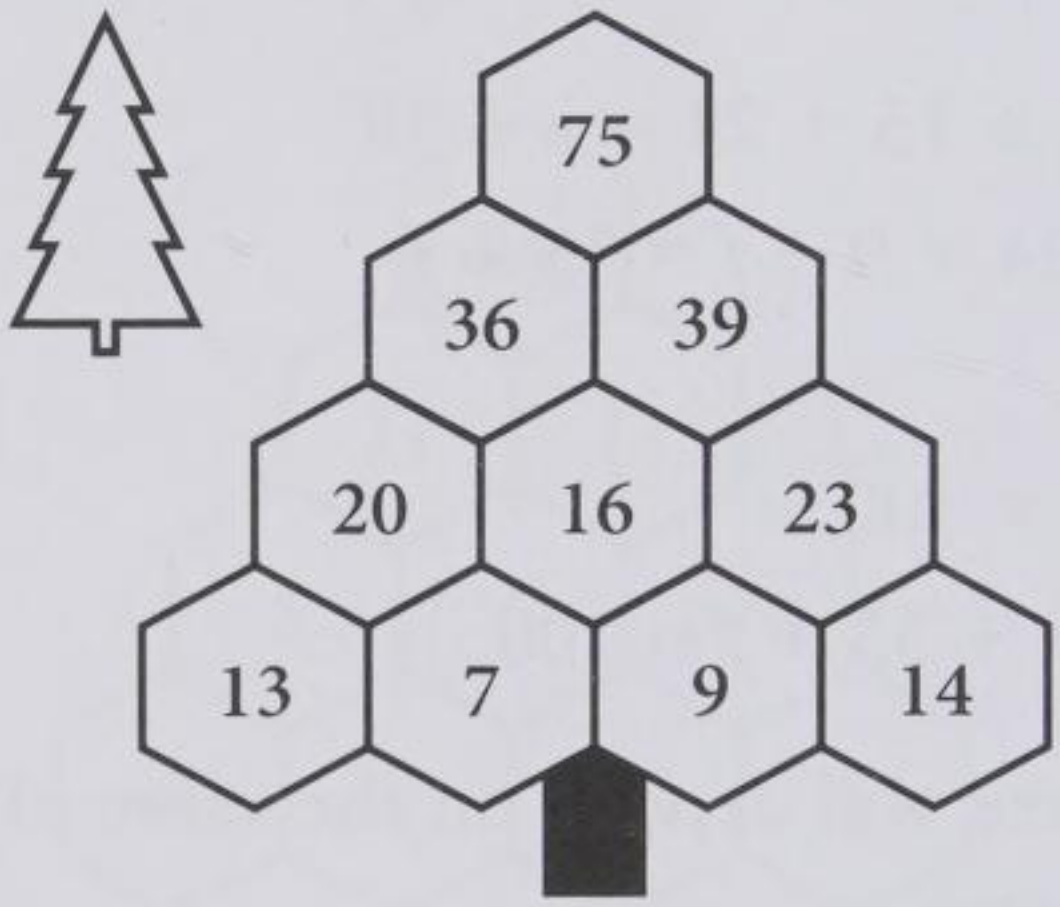
Therefore,



This is a solution. There may be many solutions for this

number tree and for different values of a and b we can get different number trees.

One solution of the first number tree is:



Friends, Parents and Grandparents

This is one interesting puzzle which you would love to solve with your friends, parents and even grandparents. The best part of this puzzle is that it requires very little calculation. This is an extended version of our earlier puzzle where we calculated the number of brothers and sisters in a family. Since here we are basically trying to find three values, we will use three variables.

How would you write 327 in the standard form?

$$327 = 3 \times 100 + 2 \times 10 + 7$$

If the number in the hundred's place is x, in the ten's place is y and in the unit's place is z, then the three-digit number $= 100x + 10y + z$

Now, call your friends and instruct them to follow these steps:

➲ Take the number of brothers that you have.

- Multiply by 2.
- Add 3.
- Multiply the previous result by 5.
- Add the number of sisters.
- Multiply by 10.
- Add the number of living grandparents.
- Subtract 150 from the result.

What number are you left with?

Let's understand this with an example.

I have 3 brothers, 1 sister and 4 grandparents.

Take the number of brothers that you have = 3.

Multiply by 2 ⇨ $3 \times 2 = 6$.

Add 3 ⇨ $6 + 3 = 9$.

Multiply the previous result by 5 ⇨ $9 \times 5 = 45$.

Add the number of sisters ⇨ $45 + 1 = 46$.

Multiply by 10 ⇨ $46 \times 10 = 460$.

Add the number of living grandparents ⇨ $460 + 4 = 464$.

Subtract 150 from the result ⇨ $464 - 150 = 314$.

As you can see, the number in the hundred's place ⇨ Number of brothers = 3

Ten's place ⇨ Sister = 1

One's place ⇨ Grandparents = 4

Do you see the algebra behind this magic trick?

Yes, this puzzle is also based on algebra and we will understand it mathematically and prove it. Let's assume that you have x brothers, y sisters and z grandparents.

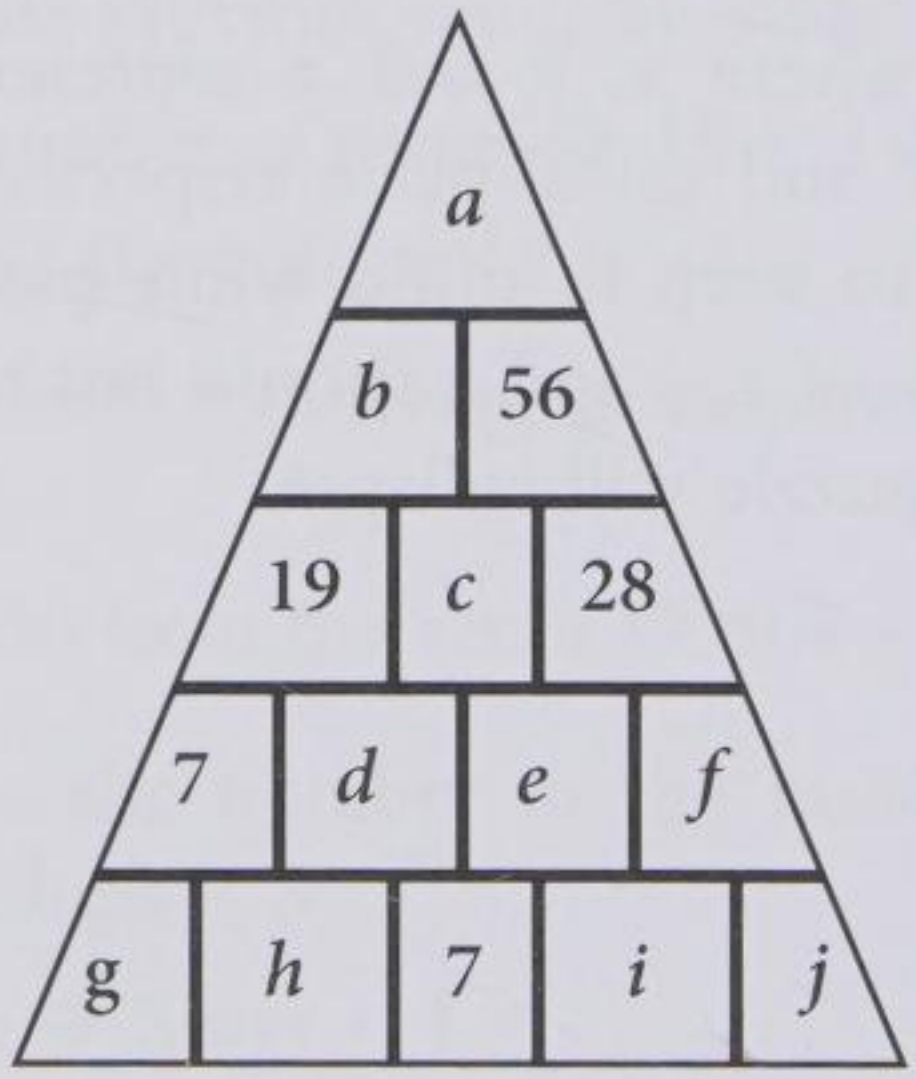
- Take the number of brothers that you have = x.
- Multiply by 2 ⇨ $2x$.
- Add 3 ⇨ $2x + 3$.
- Multiply the previous result by 5 ⇨ $5\ (2x + 3) = 10x + 15$.
- Add the number of sisters ⇨ $10x + 15 + y$.
- Multiply by 10 ⇨ $100x + 150 + 10y$.
- Add the number of grandparents ⇨ $100x + 10y + 150 + z$.
- Subtract 150 from the result ⇨ $100x + 10y + z + 150 - 150 = 100x + 10y + z$.

As you can see, at the end of the puzzle you get a three-digit number where x, y and z represent the digit at hundreds', tens' and units' place respectively. One thing that you need to keep in mind while playing this game is that the value of x, y and z should not be more than 9, otherwise this puzzle will collapse.

6

Find the Missing Number

Here is a pyramid with five layers and letters from *a* to *j* have been written inside it. In every succeeding layer, each number is the sum of the bottom two. The number at the top can be reached after the four successive bottom layers are completed. Find the value of each letter.



Solution:- The instruction at the beginning tells us that each number in the pyramid represents

the sum of the two numbers/letters immediately below it.

$7 + d = 19 \Rightarrow d = 12$

$c + 28 = 56 \Rightarrow c = 28$

$d + e = c \Rightarrow 12 + e = 28$

$\Rightarrow e = 16$

$e + f = 28 \Rightarrow 16 + f = 28$

$\Rightarrow f = 12$

$19 + c = b \Rightarrow 19 + 28 = b$

$\Rightarrow b = 47$

$b + 56 = a \Rightarrow 47 + 56 = a$

$\Rightarrow a = 103$

$h + 7 = d \Rightarrow h + 7 = 12$

$\Rightarrow h = 5$

$g + h = 7 \Rightarrow g + 5 = 7$

$\Rightarrow g = 2$

$7 + i = e \Rightarrow 7 + i = 16$

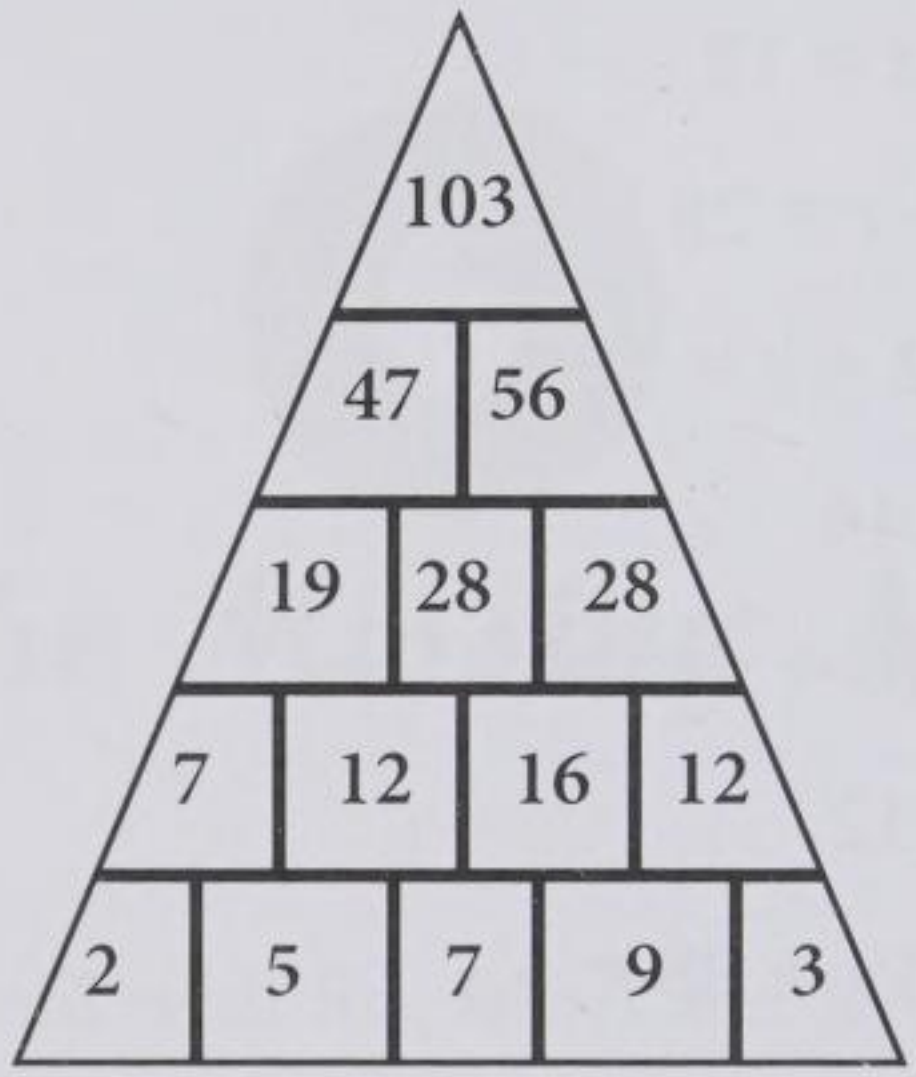
$\Rightarrow i = 9$

$i + j = f \Rightarrow 9 + j = f$

$\Rightarrow 9 + j = 12$

$\Rightarrow j = 3$

After putting these values in their respective cells, the pyramid will look like this:



7

Newton's Cross Puzzle

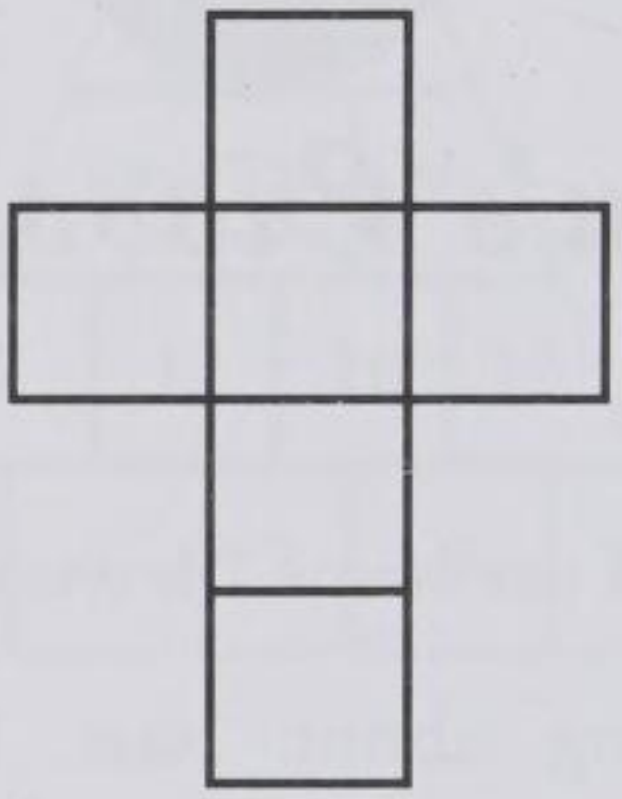
Have you heard of the name Newton?

Yes, I am talking about Isaac Newton, the great mathematician, who has been credited with the discovery of the gravitational theory in Physics when he saw an apple falling down from a tree. He is also known as the father of calculus. Newton was born on December 25, 1642, in Woolstrope, England. He was so humble that he once wrote: *I don't know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore and diverting a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.*

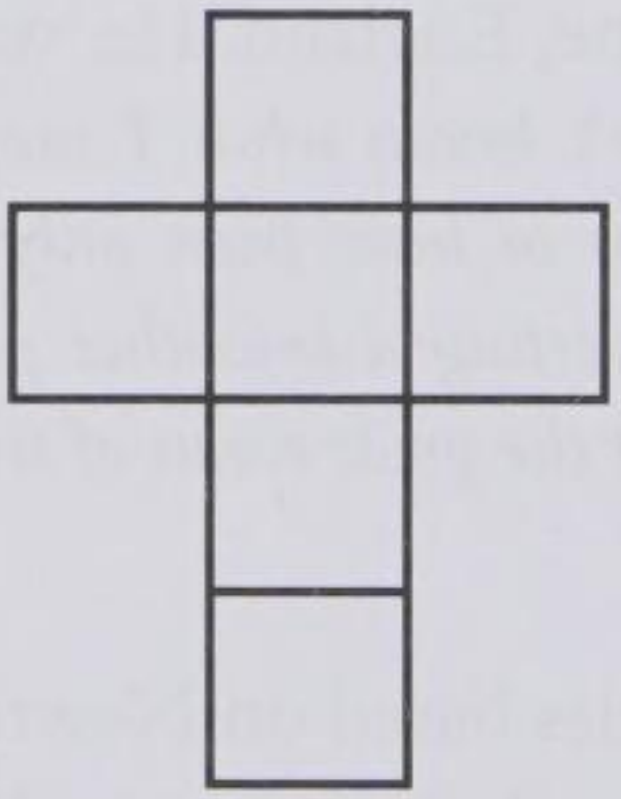
Here are two puzzles based on Newton's cross puzzle. The best part of this puzzle is its beauty. You must have solved or come across many cross puzzles where you had to put some digits in such a way that the horizontal or vertical sum of the crosses become equal. However, you would have been asked to fill only with positive numbers. This

puzzle gives you the choice of filling positive as well as negative numbers in the crosses and making the sum of horizontal and vertical crosses equal.

1. Write the numbers -6, -4, 1, 2, 3 and 5 in the correct place so that each line of the cross adds up to 3.

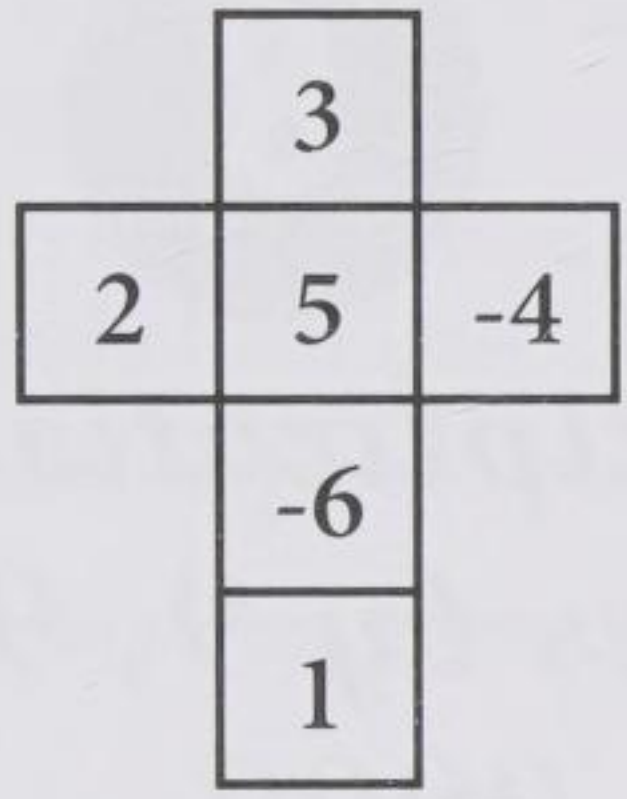


2. Write the numbers -6, -5, 2, 3, 4 and 7 in the cross so that each line of the cross adds up to 0.



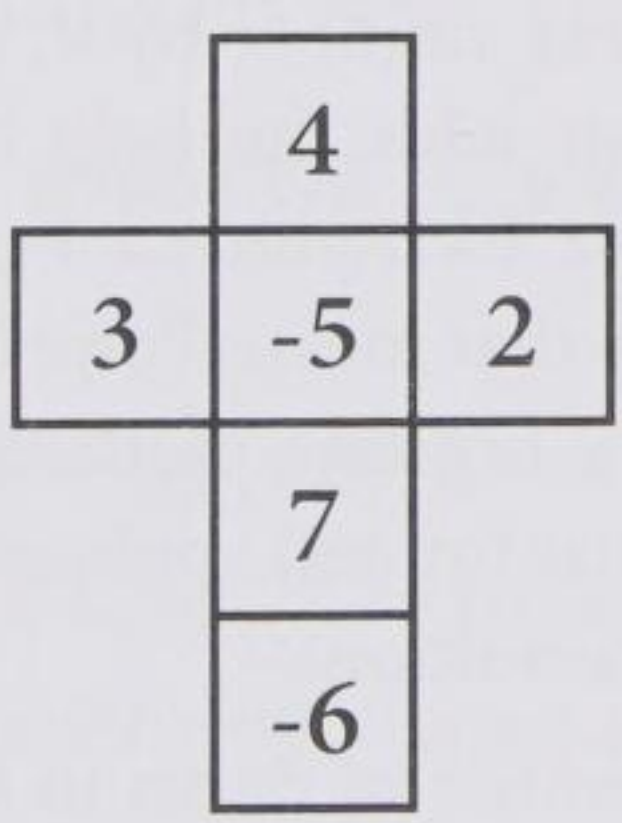
Answer: I am solving this for you. Why don't you try and find out if these crosses have a second solution?

Students must mentally calculate and place the numbers in such a way that the sum in the column and the row of each cross is the same.



Horizontal sum ⇨ 2 + 5 – 4 = 3

Vertical sum ⇨ 3 + 5 – 6 + 1 = 3



Horizontal sum ⇨ 3 – 5 + 2 = 0

Vertical sum ⇨ 4 – 5 + 7 – 6 = 0

8

Multiplication of a Number by 9, 99, 999, 9999 ...

Multiplication of any number by 9, 99, 999, 9999 ... can be done in seconds with the help of the Vedic Sutra—*Ekanyunena Purvena*. Ekanyunena Purvena is a branch of Vedic Mathematics that means "one less than the previous one". It is simply a brilliant method of multiplying two numbers, but this has limited application. This sutra works only under three conditions:-

1. When the number of digits in the multiplicand and number of 9s in multipliers are the same.
2. When the number of 9s in the multipliers is more than the number of digits in the multiplicand.
3. When there are lesser number of 9s in the multiplier than the number of digits in the multiplicand.

Now let us try each case one by one.

Case 1:When the number of digits in the multiplicand and the number of 9s in the multipliers is the same.

Rule:

- Subtract 1 from the multiplicand and write the result in LHS (left hand side).
- Subtract the LHS from the multiplier and write the result in RHS (right hand side).
- Join LHS and RHS to get the answer.

Example: Multiply 6543 by 9999.

Solution:

LHS = Multiplicand – 1 = 6543 – 1 = 6542

RHS = 9999 – 6542 = 3457

Hence, 6543 x 9999 = 65423457

Example: Multiply 89654876 by 99999999.

Solution:

LHS = 89654876 – 1 = 89654875

RHS = 99999999 – 89654875 = 10345124

Hence, 89654876 x 99999999 = 8965487510345124

Case 2:When the number of 9s in the multipliers is more than the number of digits in the multiplicand.

Rule: In case 2, the same procedure will be applied as in case 1.

Example: Multiply 456 by 9999.

Solution:

LHS = 456 – 1 = 455

RHS = 9999 – 455 = 9544

Hence, 456 x 9999 = 4559544

Example: Multiply 56892 by 9999999.

Solution:

LHS = 56892 – 1 = 56891

RHS = 9999999 – 56891 = 9943108

Hence, 56892 x 9999999 = 568919943108

Case 3: When there are lesser number of 9s in the multiplier than the number of digits in the multiplicand.

(a) Add as many zeros as the number of 9s to the multiplicand.

(b) Subtract the original multiplicand from the figure obtained in Step 1.

Example: Multiply 1564 by 99.

Solution:

The multiplicand 1564 has 4 digits, whereas there are two 9s in the multiplier.

1 5 6 4

x 9 9

1. Since there are two 9s, put two zeros at the end of 1564, making it 156400.
2. Subtract (1564 original multiplicand) from 156400.

i.e. 1 5 6 4 0 0

− 1 5 6 4

1 5 4 8 3 6

Hence, 1564 x 99 = 154836

Example: Multiply 783459 by 9999.

Solution:

1. Since there are four 9s, put four zeros at the end of 783459, making it 7834590000.
2. Subtract the original number 783459 from 7834590000.

7 8 3 4 5 9 0 0 0 0

− 7 8 3 4 5 9

7 8 3 3 8 0 6 5 4 1

Hence, 783459 x 9999 = 7833806541

This can be done by the *Ekanyunena Purvena* method effortlessly after a little practice. Let's see how it works. In this method:

(a) Subtract 1 from the original number and place the result in the LHS.

(b) Take as many number of digits, from left to right, from the multiplicand as the number of 9s in the multiplier and place it in the RHS. For example, if

you have to multiply 147 by 99, the RHS part of the equation should contain 2 digits (47) taken from right, equal to the number of 9s in the multiplier.

(c) The remaining digits in the original number, after removing the digits from the right to left, should be subtracted from the LHS.

(d) Write the complement of the digits placed in the RHS by applying the *Nikhilam sutra*. Nikhilam sutra is a Sanskrit text and is used in multiplication as well as in subtraction.

Example: Multiply 147 by 99.

Solution: Since there are two 9s in the multiplier, two digits from right to left of the multiplicand will be placed in the RHS. In the LHS, subtract 1 from the original number.

LHS = 147 – 1 = 146

RHS = Complement of 47

Now subtract the remaining digits i.e. 1 from LHS and write the complement of 47 in RHS.

The complement of any number is taken from the base digit which may be 10, 100, 1000, etc. Here the complement of 47 will be from the exponent of 10 which is greater than 47. Therefore it has to be 100.

LHS = 146 – 1 = 145

RHS = 100 – 47 = 53

Hence, 147 x 99 = 14553

Example: Multiply 259648 by 9999.

Solution: Since there are four 9s in the multiplier, four digits from right to left of multiplicand i.e. 9648 will be placed in the RHS. In LHS, subtract 1 from the original number.

LHS = 259648 – 1 = 259647

RHS = Complement of 9648

Now subtract the remaining digits i.e. 25 from LHS and write the complement of 9648 in RHS.

LHS = 259647 – 25 = 259622

RHS = 10000 – 9648 = 0352

Hence, 259648 x 9999 = 2596220352

Palindrome Number

A palindrome is a word, sentence, verse or number that reads the same forward and backward. The word palindrome comes from the Greek word *palindromus*, which means to run back again.

11, 121, 14641 etc. are all palindromes.

Now, the question is how to make a palindrome number.

Example:-

- Take any two-digit number.
- Reverse the digits of the original number.
- Add the new number to the original number.

Did you get a palindrome number?

If not, then repeat the process once again, and you are done.

Example: Let the two-digit number = 67

Reverse the digits ⇨ 76

Add the numbers ⇨ 67 + 76 = 143

Reverse the digit ⇨ 341

Add the numbers ⇨ 143 + 341 = 484

Remember: Don't take the number 196. A computer has already gone through several thousand stages and still hasn't come up with a palindrome answer using it!

Indian mathematician Mahavira has discussed palindrome numbers in his book *Ganit Sar Sangraha*.

Let's see a few examples of palindrome numbers here.

$3003 \times 37 = 111111$

$6006 \times 37 = 222222$

$9009 \times 37 = 333333$

$12012 \times 37 = 444444$

$15015 \times 37 = 555555$

$18018 \times 37 = 666666$

$21021 \times 37 = 777777$

$24024 \times 37 = 888888$

$27027 \times 37 = 999999$

$152207 \times 73 = 11111111$

$14287143 \times 7 = 10010001$

$12345679 \times 9 = 111111111$

$142857143 \times 7 = 1000000001$

$11011011 \times 91 = 1002002001$

$27994681 \times 441 = 12345654321$

Can you give me a few prime numbers that are palindromes?

They are - 11, 101, 131, 151 ...

Try to find some more.

There are many square palindrome also.

11 is a palindrome number and its square 121 is also a palindrome.

22 is a palindrome, so its square is 484.

There are 9 palindrome of 2 digits and 90 palindromes of 3 digits. You can explore this new knowledge further.

The first 9 terms of the sequence

$1^2, 11^2, 111^2, 1111^2$ ... are also palindromes.

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1.234321$.

Can you find some cubes that are palindromes?

Is the cube of 7 a palindrome?

Yes , $7^3 = 343$, a palindrome.

Find some more numbers raised to the power four that are palindromes.

10

Magical Number 1089

You can play this interesting game with many friends simultaneously during the lunch break in school or at a birthday party and attract the attention of all your friends.

- Take some pieces of paper and write 1089 on each of them.
- Fold and put them in a box.
- Tell your friends to think of any three-digit number, but the digit at the hundred's place should be more than the digit placed at the unit's place.
- Reverse the order of the number.
- Take the difference between the original number and the reverse number.
- Reverse the number obtained in the previous step and sum up both the numbers.

Example 1:

Take a three-digit number = 421

Reverse it = 124

Find the difference	= 421 – 124 = 297
Reverse the number	= 792
Add the two numbers	= 297 + 792 = 1089

Example 2:

Take a three-digit number	= 987
Reverse it	= 789
Find the difference	= 987 – 789 = 198
Reverse the number	= 891
Add the two numbers	= 891 + 198 = 1089

Tell your friends to stop as the game is over.

Call your friends one by one and tell them to pick out a piece of paper from the jar. Tell them to read the number written on their chit, one by one. They will be astonished to know that you would have predicted the result well in advance.

Is it true for all three-digit numbers?

Yes, this game is valid for all three-digit numbers, provided all the digits of the number are different and the digit at the hundred's place is larger than the digit at the unit's place.

Let the three-digit number be $100x + 10y + z$ where x = digit at the hundred's place, y = digit at the ten's place and z = digit at one's place.

	Hundreds	Tens	Ones
Number	x	y	z
Reverse number	z	y	x

(Subtract, considering $x > z$)

	Hundreds	Tens	Ones
Number	$x - 1$	$y + 9$	$z + 10$
Reverse number	z	y	x
Subtract	$x - 1 - z$	9	$z + 10 - x$
Reverse number	$z + 10 - x$	9	$x - 1 - z$
	9	18	9

The answer = 1089.

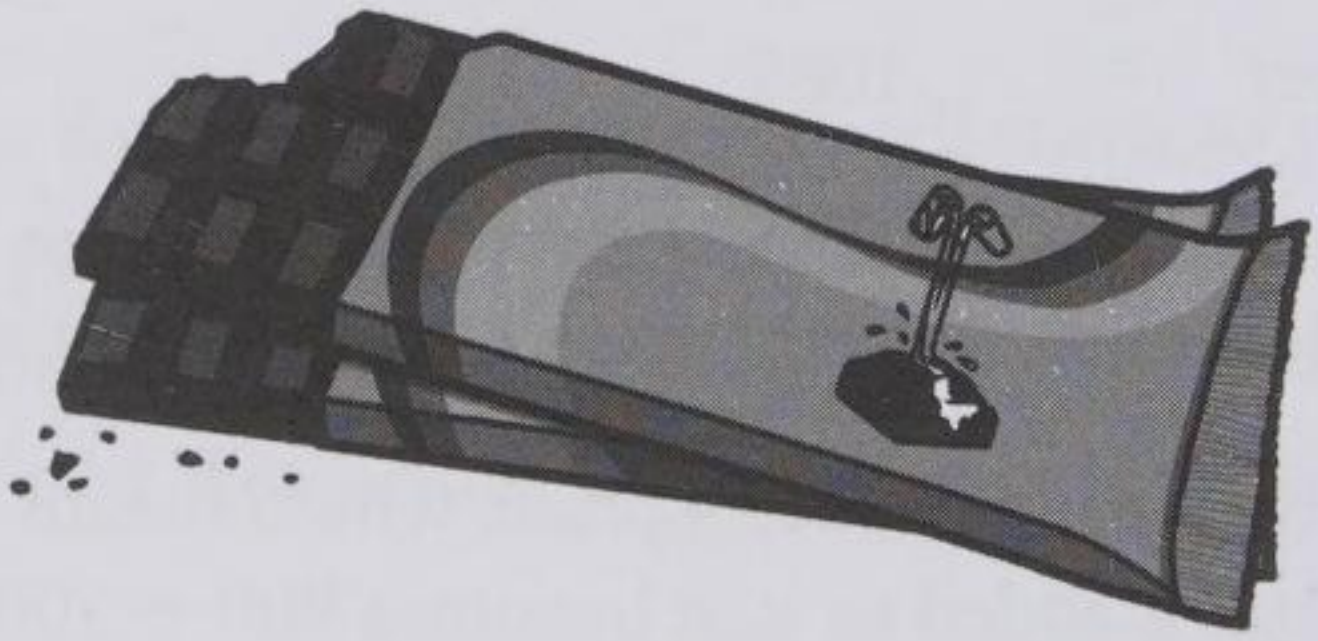
Here 18 tens = 180, so 8 is kept at ten's place and 1 is transfered to the hundred's place, making 9 as $9 + 1 = 10$.

9 is already in the hundred's place, which means it is 900, so when 100 is added to it, it becomes $900 + 100 = 1000$.

Therefore, $1000 + 80 + 9 = 1089$.

11

Eat Chocolate and Tell People's Age



It has been medically proved that chocolate helps in enhancing our ability to calculate. Professor David Kennedy, director of the Brain, Performance and Nutrition Research Centre at Northumbria University, and co-author of the study, said that chocolate could be beneficial for mentally challenging tasks.

You will probably laugh if I say that I can guess your age if you agree to do some calculations and tell me the

number of times you crunch a chocolate bar. Yes, it's true. Mathematics can validate this silly-looking puzzle.

So, are you ready to play this game with me?

Take your pen, pencil and calculator and follow the steps.

- Write how many times you eat or want to eat chocolate in a week. Any number between 1 to 100 is valid.
- Multiply it by 2
- Add 5
- Multiply the previous result by 50.
- Add the current year (2017, 2018 ...).
- If your birthday has gone by, then subtract 250, else subtract 251.
- Now subtract your birth year from the previous result.
- You will end up with a three-digit or a four-digit number.

The last two digits of the result will tell your age and the remaining digits will tell how many times in a week you crunch chocolates.

Let's take an example.

Example: My son eats chocolate 6 times in a week and he was born on April 5, 2010.

Let's help him play this game and see how he does the calculations.

- Write how many times you eat or want to eat chocolate in a week = 6.
- Multiply it by 2 ⇨ 6 × 2 =12
- Add 5 ⇨ 12 + 5 = 17
- Multiply the previous result by 50 ⇨ 17 × 50 = 850
- Add the current year (2017, 2018 ...) ⇨ 850 + 2017= 2867
- If your birthday has gone by then subtract 250, else subtract 251 = 2867 – 250 = 2617
- Now subtract your birth year from the previous result = 2617 – 2010 = 607

Here, the last two digits = 07 = Age

Remaining digit = 6 = Number of times he eats chocolate.

Can you do it algebraically?

- Write how many times you eat or want to eat chocolate in a week = y.
- Multiply it by 2 ⇨ $2y$
- Add 5 ⇨ $2y + 5$
- Multiply the previous result by 50 ⇨ $100y + 250$
- Add the current year (2016, 2017 ...) ⇨ $100y + 250 + 2017 = 100y + 2267$
- If your birthday has gone by, then subtract 250, else subtract 251.

  ⇨ $100y + 2267 – 250 = 100y + 2017$.

(Here you can see that the number of times you eat

chocolate is multiplied by 100 and then the current year is added.)

➲ Now subtract your birth year from the previous result ⇨ $100y + 2017 - 2010 = 100y + 07$.

This will be a three or four-digit number whose last two digits will tell you the age and the first value of y. The value of y tells you the number of times you eat chocolate in a week.

If $y = 6$,

$100y + 7 = 607$

Here the last two digits = 07 = Age

Remaining digit = 6 = Number of times he eats chocolate.

12

Number Through Remainder

This is an interesting game where the remainder will help find out a number you have thought of. In simple words, I can tell you the number you have thought of if you give me the remainder after each division. Take a pen and a piece of paper and get ready. There is no need to take a calculator as you won't get a remainder when dividing with a calculator.

You all know:

Dividend = Divisor × Quotient + Remainder

Here, I shall take the help of this formula and some magical tricks to find the original number that you took at the beginning.

- Take any number between 6 and 60.
- Divide it by 3 and give me the remainder. Let the remainder in this case be *a*.

- Divide the number by 4 and give me the remainder. Let the remainder be b.
- Divide the number by 5 and give me the remainder. Let the remainder be c.

I bet that I can guess the number you have thought of at the beginning of the game correctly.

I shall simply put the value of a, b and c in $40a + 45b + 36c$ and divide it by 60. The remainder in this case will be the number taken at the very beginning of the game.

Example:

Number = 53

Divide it by 3 ⇨ $53 \div 3$ R = 2 (say a)

Divide it by 4 ⇨ $53 \div 4$ R = 1 (say b)

Divide it by 5 ⇨ $53 \div 5$ R = 3 (say c)

Place the value of a, b and c in the given expression: $40a + 45b + 36c$

$$= 40 \times 2 + 45 \times 1 + 36 \times 3$$

$$= 80 + 45 + 108$$

$$= 233$$

Divide the remainder obtained at the end by 60 = $233 \div 60$

Here Q = 3 and R = 53

In case your remainder = 0, then your chosen number at the beginning = 60.

Example:- Number = 60

Divide it by 3 ⇨ $60 \div 3$ R = 0 (say *a*)

Divide it by 4 ⇨ $60 \div 4$ Q = 0 (say *b*)

Divide it by 5 ⇨ $60 \div 5$ Q = 0 (say *c*)

Place the value of *a*, *b* and *c* in the given expression:
$40a + 45b + 36c = 40 \times 0 + 45 \times 0 + 36 \times 0 = 0$

Divide the remainder obtained at the end by $60 = 0 \div 60$

Here Q = 60 and R = 0

NB:- Here the LCM of 3, 4 and 5 = 60 is taken at the end to divide a number.

13

Guessing Three Consecutive Numbers

This puzzle will let you guess the three consecutive numbers thought of by your friend while playing a mathematical game.

Give a pen, paper and calculator to your friend and tell him or her to do the following mathematical operations:

- Choose three consecutive numbers of 2 digits.
- Choose a multiple of 3.
- Add all four numbers.
- Multiply it by 67.
- Give me the last two digits of your final result and I will give you the three numbers you have chosen.

Example:

- Let the three consecutive numbers be 65, 66 and 67. Multiple of 3 that you want to add = 99.
- Add the four numbers ⇨ 65 + 66 + 67 + 99 = 297.

- Multiply it by 67 ⇨ 297 × 67 = 19899.
- Last two digits of your product = 99.

Now it is my turn to tell you the number you have taken.

But let's perform two steps with your numbers before we get to the number.

Step 1:- I will simply divide the multiple of 3 by 3 and add 1 to it.

$99 \div 3 = 33$

$33 + 1 = 34$

Step 2:- Subtract 34 from the last two digits of the answer.

$99 - 34 = 65 = 1^{st}$ number

$65 + 1 = 66 = 2^{nd}$ number

$66 + 1 = 67 = 3^{rd}$ number

Let me give you another example so that you understand better.

- Let the three consecutive numbers be 38, 39 and 40..
- Multiple of 3 that you want to add = 27.
- Add the four numbers ⇨ 38 + 39 + 40 + 27 = 144.
- Multiply it by 67 ⇨ 144 × 67 = 9648.

Last two digits of your product = 48.

In order to get the three consecutive numbers, follow these steps:

Divide the multiple of 3 by 3 and add 1 to it ⇨
27/3 + 1 = 10.

Subtract the result from the last two-digit answer ⇨ $48 - 10 = 38$.

Hence,

1^{st} number = 38

2^{nd} number = 39

3^{rd} number = 40

Can you guess how I know the answer?

This is the magic of mathematics.

Let the three consecutive numbers be ⇨ $a, a + 1$ and $a + 2$

Multiple of $3 = 3k$

Add them ⇨ $a + a + 1 + a + 2 + 3k = 3a + 3 + 3k$

Multiply it by 67 ⇨ $67 \times (3a + 3k + 3) = 201\ (a + k + 1)$

This shows that the product obtained is the multiple of 201.

Hence the number = $a + k + 1$

Multiple of 3 ⇨ $3k$

Divide it by 3 and add 1 ⇨ $3k/3 + 1 = k + 1$

Subtract $k + 1$ from the last two digits of the result ⇨ $a + k + 1 - k - 1 = a$

Therefore, 1^{st} number = a

2^{nd} number = $a + 1$

3^{rd} number = $a + 2$

14

Difference of Two Squares Puzzle

You must have read the formula:

$A^2 - B^2 = (A + B)\ (A - B)$

This puzzle is based on the above formula. Once you learn the technique of this puzzle you will definitely love to play this game with your friends. Moreover, at the end of the game I shall tell you the secret of squaring a two-digit number in a few seconds. So get ready with a pen, paper and calculator.

- Choose any two-digit number.
- Multiply it by 6.
- Square the number initially taken.
- Add the previous two results.
- Now add 9 to it.
- Finally subtract the square of the number initially taken from the previous result.

If you tell me the number chosen by you then I shall tell you the final result obtained.

Let's first take an example.

Example:

- Choose any two-digit number = 18
- Multiply it by 6 ⇨ $18 \times 6 = 108$
- Square the number initially taken ⇨ $18^2 = 324$
- Add the previous two results ⇨ $324 + 108 = 432$
- Now add 9 to it ⇨ $432 + 9 = 441$
- Finally subtract the square of the number taken from the previous result ⇨ $441 - 18^2$

  $= 441 - 324$

  $= 117$

I had promised you that if you tell me the number taken by you at the beginning of the game, I shall tell you the final result obtained.

The trick = $6 \times$ number + 9

$= 6 \times 18 + 9 = 117$

Let's understand the secret of this game with simple algebra.

- Choose any two-digit number = y.
- Multiply it by 6 ⇨ $6y$.
- Square the number initially taken ⇨ y^2.
- Add the previous two results ⇨ $y^2 + 6y$.
- Now add 9 to it ⇨ $y^2 + 6y + 9$.

➲ Finally subtract the square of the number taken from the previous result

⇨ $y^2 + 6y + 9 - y^2 = 6y + 9$.

As you have seen here, puzzles of this type require squaring a number. Let's learn a few tips on squaring that will boost your calculating ability.

Squaring a two-digit number

You must be familiar with the formula

$$A^2 = (A + D)\ (A - D) + D^2$$

Let's use the above formula in squaring a number. In the above formula, A is the number to be squared and D is the number that makes one of the pair of A in the multiple of 10. That is to say, we know that

$(a + b)\ (a - b) = a^2 - b^2$

Here, $(A + D)\ (A - D) = A^2 - D^2$

Hence,

$A^2 - D^2 + D^2 = A^2$

Example: Find the square of 14

Solution:

$$14^2 = 14 \begin{cases} +4 \\ -4 \end{cases} \Big\rangle \begin{matrix} 18 \\ | \ x \\ 10 \end{matrix} \quad 180 + 4 \times 4 = 196$$

Example: Find the square of 36

Solution:

$$36^2 = 36 \begin{array}{l} \nearrow\ +6 \\ \searrow\ -6 \end{array} \begin{array}{c} 42 \\ \text{x} \mid \\ 30 \end{array} \quad 1260 + 6^2 = 1296$$

Example: Find the square of 84

Solution:

$$84^2 = 84 \begin{array}{l} \nearrow\ +4 \\ \searrow\ -4 \end{array} \begin{array}{c} 88 \\ \mid \\ 80 \end{array} \quad 7040 + 4^2 = 7056$$

I hope you will love squaring with the unconventional method discussed above. You can extend the same method for squaring a three or four-digit number.

Example: Find the square of 187.

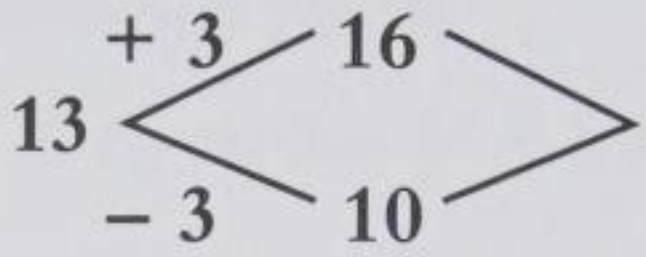
Solution: Adding and subtracting 7 will make the product of two numbers difficult.

$$187^2 = 187 \begin{array}{l} \nearrow\ +7 \\ \searrow\ -7 \end{array} \begin{array}{c} 194 \\ \text{x} \mid \\ 180 \end{array}$$

Let's think of another combination that can make our calculation easier.

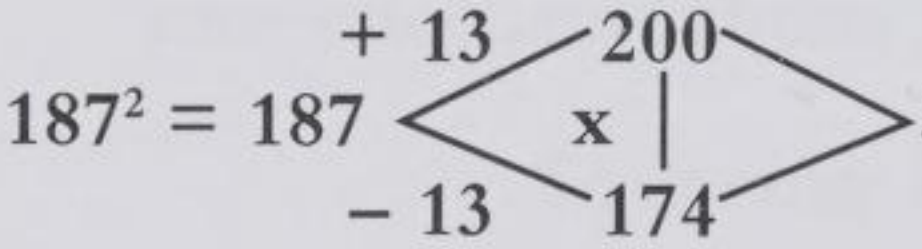
$$187^2 = 187 \begin{array}{l} \nearrow\ +13 \\ \searrow\ -13 \end{array} \begin{array}{c} 200 \\ \text{x} \mid \\ 174 \end{array} \quad 34800 + 13^2$$

The square of 13 can be obtained in this manner.



$$160 + 3^2 = 169$$

Hence,



$$34800 + 13^2 = 34800 + 169 = 34969$$

15

Odd-Even Alphametics

Alphametics!

Don't look up this word in the dictionary.

Alphametics is the combination of two words—Alphabetic + Arithmetic. When combined in the form of a mathematical puzzle, a new word was born in 1955 known as *alphametics*.

In simple words, it is a mathematical puzzle consisting of a numerical computation with letters substituted for numbers which are to be restored through mathematical reasoning. In Alphametics, letters are interchanged with numbers

What is odd and even?

As far as the definition of odd and even in mathematics is concerned, a number that ends with 1, 3, 5, 7 or 9 is known as an odd number whereas a number that ends with 0, 2, 4, 6 or 8 is known as an even number.

Have you heard about the odd-even scheme initiated by the Delhi government to reduce pollution levels in Delhi?

The scheme aimed to reduce pollution levels by allowing cars with even registration numbers to ply on even dates and the ones with odd numbers to ply on odd dates.

This puzzle is dedicated to the Delhi government for their bold step.

Question:

$$\begin{array}{r} O\,E\,E \\ \times \quad E\,E \\ \hline E\,O\,E\,E \\ E\,O\,E\,\times \\ \hline O\,O\,E\,E \end{array}$$

Here E = even number and O = odd number

Replace E with even digits and O with odd digits. The digits may change.

Solution:

$$\begin{array}{r} A\,B\,C \\ \times \quad D\,Z \\ \hline F\,G\,H\,I \\ K\,L\,M \\ \hline N\,P\,Q\,R \end{array}$$

(i) $A\,B\,C \times Z = F\,G\,H\,I$

(ii) $A\,B\,C \times D = K\,L\,M$

(iii) $A\,B\,C \times D\,Z = N\,P\,Q\,R$

(iv) $F\,G\,H\,I + K\,L\,M = N\,P\,Q\,R$

D is an even number, so $D \geq 2$

Hence, $A \times D = K \leq 9$, otherwise $F + K > 9$ which will make the equation wrong. Moreover, D is an even number (2, 4, 6, 8), so it is a multiple of 2. So, from equation (ii), A will be $\leq 9/2$. Hence $A = 1$ or 3.

If $A = 1$ then $A\ B\ C < 200$, so $ABC \times Z < 2000$, but F is an even number, so $F = 1$ contradicts the facts. So A is obviously 3.

Equation (ii) implies $A \times D = K$ which is less than 9 as shown above.

Hence, $D = 2$ is the only option left. For $D = 4$ or 6 or 8, we will never get $3 \times D < 9$

(i) $A\ B\ C \times Z = F\ G\ H\ I$ (= EOEE) ≥ 2100

$3\ B\ C \times Z \geq 2100$

So Z will be either 6 or 8

300 x 8 = 2400 and 3BC x 6 may be greater than 2100 if BC takes a value more than 50.

Now we have $A = 3$, $Z = 6$ or 8 and $D = 2$.

Look at the different combinations now

326 x 26 = 8476

328 x 26 = 8528

346 x 26 = 8996

348 x 26 = 9048

Again,

326 x 28 = 9128

328 x 28 = 9184

None of these fit in the OOEE (Final Answer) combination.

346 x 28 = 9688

348 x 28 = 9744

In the above case, three combinations except 346 x 28 = 9688 fit the final answer OOEE.

Let's check which one is the right answer.

326 x 8 = 2608

328 x 8 = 2624

346 x 8 = 2768

348 x 8 = 2784

Now we can see that only

346 x 8 = 2768 and

348 x 8 = 2784 fit the EOEE combination of equation (i). Out of these two, only 348 x 28 = 9744 is fit for our final answer OOEE. Hence,

$$\begin{array}{r} 348 \\ \times\ 28 \\ \hline 2784 \\ 696\times \\ \hline 9744 \end{array}$$

is the final answer.

16

Unique Multiplication

There is a unique characteristic associated with numbers 46 and 96; if you change the positions of the digits, the product of the new numbers is the same as that of the old ones.

46 x 96 = 4416

64 x 69 = 4416

There are several pairs of numbers that can yield such a result. Can you help find such pairs of numbers?

Solution:

The solution of this problem can be arrived at with the help of algebra.

Let the digits of a two-digit number be x and y.

Number = $10x + y$

Reverse number = $10y + x$

Again, let the digits of another set of two-digit numbers be z and t.

Number = $10z + t$

Reverse number = $10t + z$

According to the above example,

46×96	$= 64 \times 69$
$(10x + y) \times (10z + t)$	$= (10y + x) \times (10t + z)$
$\Rightarrow 100xz + 10xt + 10zy + ty$	$= 100yt + 10yz + 10tx + xz$
$\Rightarrow 100xz + ty$	$= 100ty + xz$
$\Rightarrow 99xz$	$= 99yt$
$\Rightarrow xz$	$= yt$

Here x, y, z and t are all whole numbers less than 10. Hence, their possible pairs are:

$1 \times 4 = 2 \times 2$

$1 \times 6 = 2 \times 3$

$1 \times 8 = 2 \times 4$

$1 \times 9 = 3 \times 3$

$2 \times 6 = 4 \times 3$

$2 \times 8 = 4 \times 4$

$2 \times 9 = 6 \times 3$

$3 \times 8 = 4 \times 6$

$4 \times 9 = 6 \times 6$

For pair of product: $1 \times 4 = 2 \times 2$, we can write

$12 \times 42 = 21 \times 24$

1 x 4 = xz and 2 x 2 = yt. Therefore, xy = 12 and zt = 42

For pair of product:- $1 \times 6 = 2 \times 3$, we can write $12 \times 63 = 21 \times 36$

or $13 \times 62 = 31 \times 26$

Hence the possible pairs are:

$$12 \times 24 = 21 \times 24$$

$$12 \times 63 = 21 \times 36$$

$$12 \times 84 = 21 \times 48$$

$$13 \times 62 = 31 \times 26$$

$$13 \times 93 = 31 \times 39$$

$$14 \times 82 = 41 \times 28$$

$$23 \times 64 = 32 \times 46$$

$$23 \times 96 = 32 \times 69$$

$$24 \times 63 = 42 \times 36$$

$$26 \times 93 = 62 \times 39$$

$$34 \times 84 = 43 \times 68$$

$$36 \times 84 = 63 \times 48$$

$$46 \times 96 = 64 \times 69$$

17

Mathematical Word Play

In this game, letters are rearranged in such a way that a word or phrase is formed. Here are a few such words; find out what these mathematical words are.

ANGEL	RENUMB	CAR	DDA	CAGEDON
MOLERKITE	DOD	NISE	PELILSE	TGAENNT
NEVE	ACROFT	LIBLION	IEMT	HESPRE
SMU	DREAMISTE	CHIN	GRITH	SENT
LOPES	CLEANSE	PHASE	TRANSEPT	PITON

Solution:

1st row

1. ANGLE 2. NUMBER 3. ARC 4. ADD 5. DECAGON

2nd row

1. KILOMETER 2. ODD 3. SINE 4. ELLIPSE 5. TANGENT

3rd row

1. EVEN 2. FACTOR 3. BILLION 4. TIME 5. SPHERE

4th row

1. SUM 2. DIAMETERS 3. INCH 4. RIGHT 5. TENS

5th row

1. SLOPE 2. SCALENE 3. SHAPE 4. PATTERNS 5. POINT

18

Kaprekar Constant

Have you ever heard about the **Kaprekar Constant**?

Kaprekar Constant was named after an Indian mathematician Dattaraya Ramachandra Kaprekar (1905-1986) who was a recreational mathematician. He was born on January 17, 1905 in Maharashtra and is better known for his discovery of the Kaprekar number, the Harshad number and self numbers.

Do you know?

6174 is known as the Kaprekar Constant.

Call some of your friends to play this beautiful game on

Kaprekar Constant. Before that, write 6174 on your hand with soap cake. When you write anything on your hand with wet soap, it will not appear instantly. To get the result, 6174 is written with wet soap which will not be visible until it is rubbed by ash. Give a pen, paper and calculator to your friends and instruct them to follow these steps:

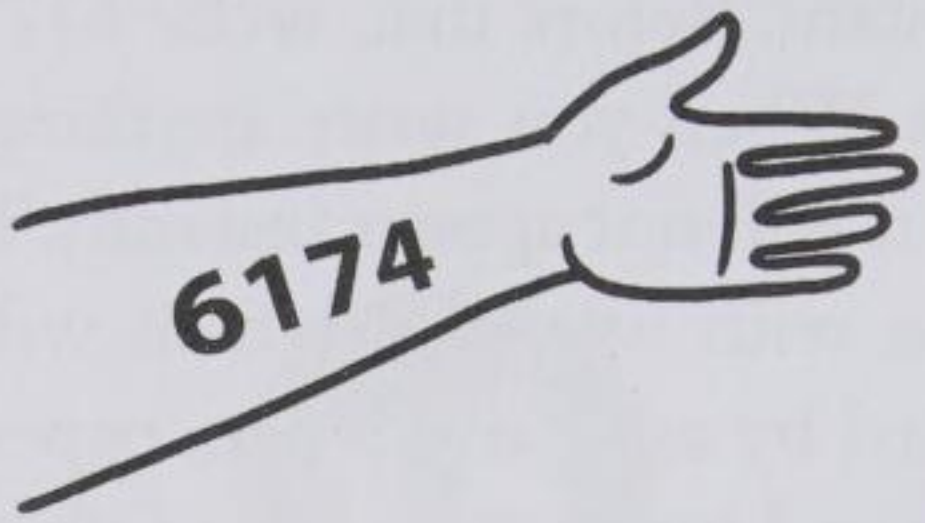
- Take any four-digit number, using at least two different digits. It will be better if all four digits are different from each other.
- Arrange the digits in descending and then in ascending order to get two four-digit numbers.
- Subtract the smaller number from the bigger one.
- Repeat the process until you get a constant number. A constant number is a number that is fixed, and after any operation, the result will not be changed. Here 6174 will be the answer, and after any further operations as directed, the result will still be 6174.
- Stop.

Once your friends stop and say that they have reached a constant number, tell them to write the constant number on a slip of paper and put it in a jar next to them.

Put the slip of paper on fire and keep it in the jar. Put the ashes on your hand.

You will get the answer as 6174 appears on your hand. That is enough to prove your magical-mathematical ability to your friends.

Caution: This game is not to be played without parental guidance.



Example: Let's take 3542 to see this magic.

Arrange digits in descending order ⇨ 5432.

Arrange them in ascending order ⇨ 2345.

Subtract the smaller number from the larger one ⇨ 5432 – 2345 = 3087.

Arrange digits in descending order ⇨ 8730.

Arrange them in ascending order = 0378.

Subtract the smaller number from the larger one ⇨ 8730 – 0378 = 8352.

Arrange digits in descending order = 8532.

Arrange them in ascending order = 2358.

Subtract the smaller number from the larger one ⇨ 8532 – 2358 = 6174.

Arrange digits in descending order = 7641.

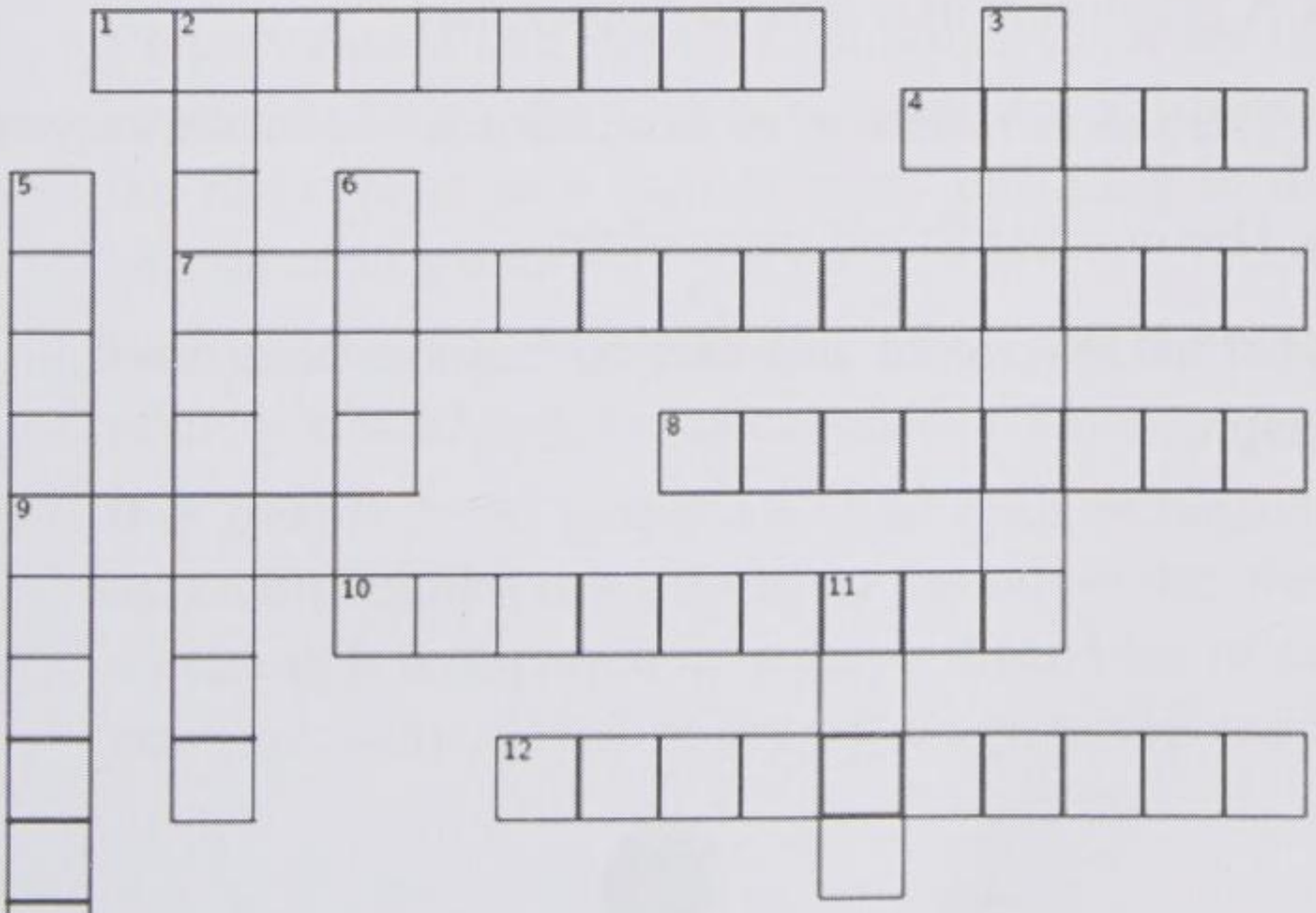
Arrange them in ascending order = 1467.

Subtract the smaller number from the larger one ⇨ 7641 – 1467 = **6174.**

19

Mathematical Crossword

This puzzle will test your maths vocabulary and your skill to guess the correct mathematical word based on its definition. So let's test your mathematical brain. You can see the vertical and horizontal bars with empty boxes here that have to be filled with the instruction provided here. There are 12 words to be filled in, and at the end I shall explain the word with a proper definition wherever necessary.



Across

1. Any mathematical process.

4. The lowest value in a set of numbers upto the highest value in the set.

7. A network of lines used for locating points.

8. Any symbol that could represent a number.

9. Average

10. Graph that displays data using line segments.

12. A model or drawing based on a ratio.

Down

2. The positions of a single digit in the whole number.

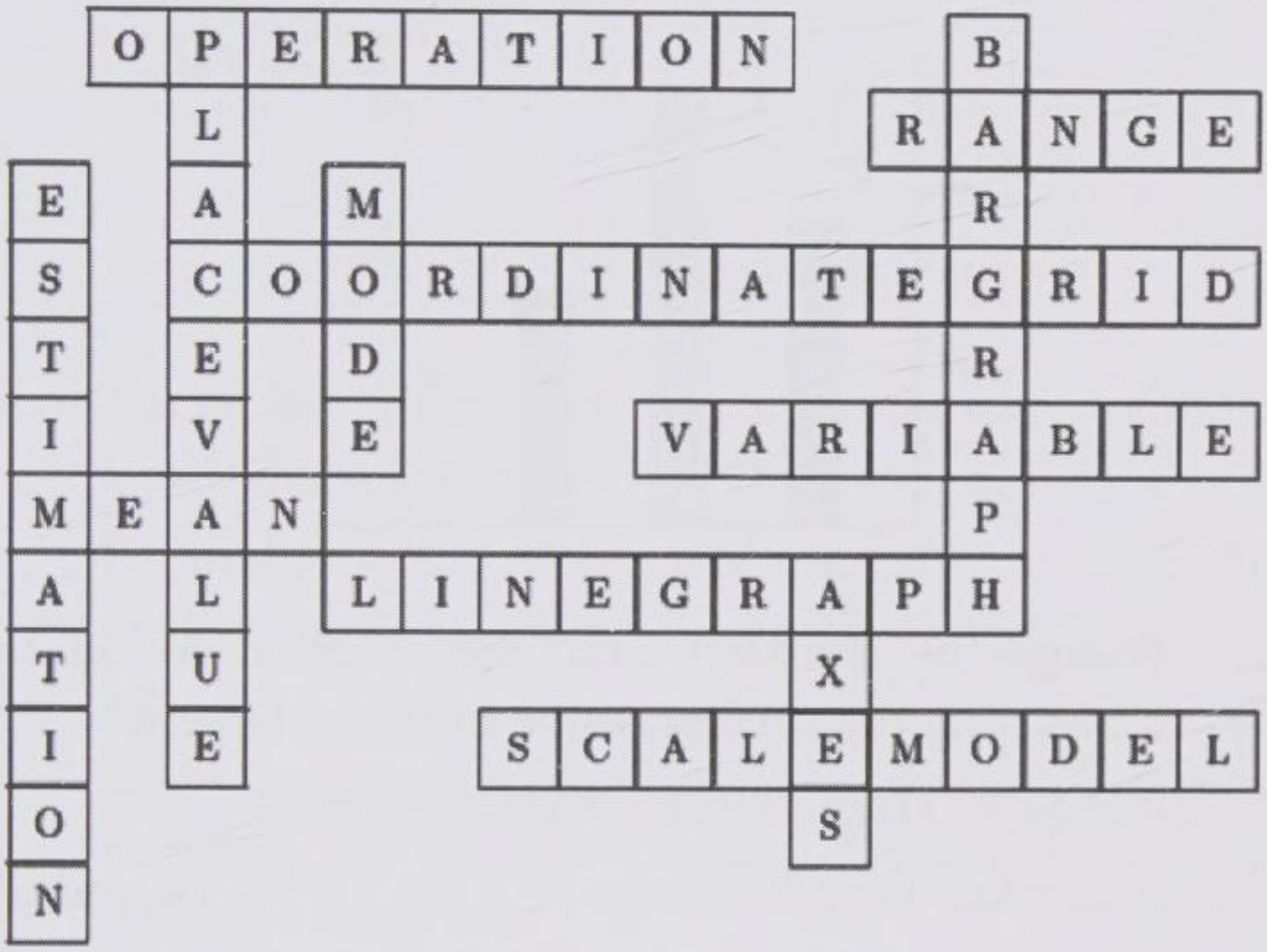
3. A graph that uses bars to display data.

5. The use of rounding to determine a reasonable answer.

6. The number found most often.

11. The horizontal and vertical number lines used in a graph.

Solution:-



Explanation:-

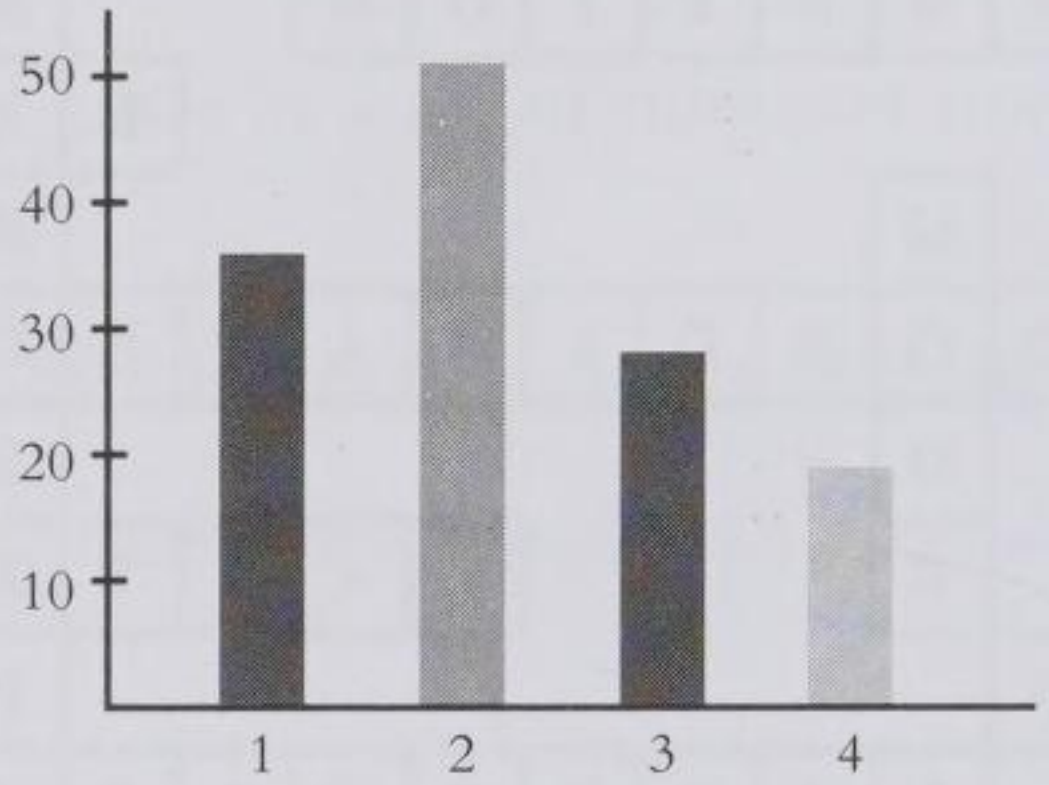
1. **Operations**: Plus, minus, multiplication and division are known as the four fundamental operations, in mathematics, .

2. **Place value**: Place value of a number expresses the position of that digit in a number. In a numeral, the place value of a digit changes according to the change of its place.

   Example: In 764, the place value of 7 is 700, the place value of 6 is 60 and the place value of 4 is 4.

3. **Bar graph**: A bar graph is a chart with rectangular bars with length proportional to the values that they represent. It is displayed on a graph sheet. One of the axes (normally *x*-axis) of the bar diagram represents

a discrete variable while the other axis (y-axis) represents the scale for continuous variable.



4. **Range**: In statistical data, the range is found by subtracting the smallest value from the largest.

   Range ⇨ Highest value – Smallest value

   Example:- Find the range of 7, 24, 14, 54, 12, and 61

   Solution:- Range = 61 – 7 = 54

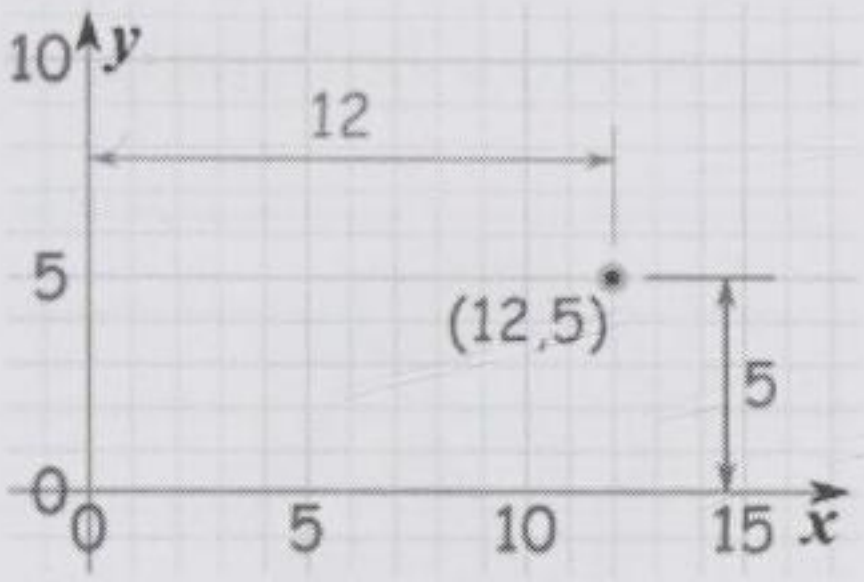
5. **Estimation**: A close guess of actual value, usually with some thought or calculation involved.

   Example:- You can estimate that there are 5000 buses plying on the Delhi's roads.

6. **Mode**: The number which appears most often in a set of numbers.

   Example:- In {6, 3, 9, 6, 6, 5, 9, 3} the mode is 6 as it occurs the most number of times.

7. **Co-ordinate**: A branch of geometry where a set of points is represented on a plane. The plane contains the x-axis and the y-axis. The point represented shows its distance from the respective axes as shown below.



8. **Variable**: A symbol of an unknown number. Generally we represent variables with x, y, z ...

   Example:- In $2x + 4$, x is the variable and 2 is the coefficient of variable. 4 is termed as constant.

9. **Mean**: The mean of a set of numbers is also known as the average. To calculate the mean of a group of numbers, just add them up and divide by how many numbers there are.

   Example: Calculate the mean of 15, 13, 18, 16, 14, 17 and 12

Solution:

$$\text{Mean} = \frac{\text{Sum of all data values}}{\text{Number of data values}}$$

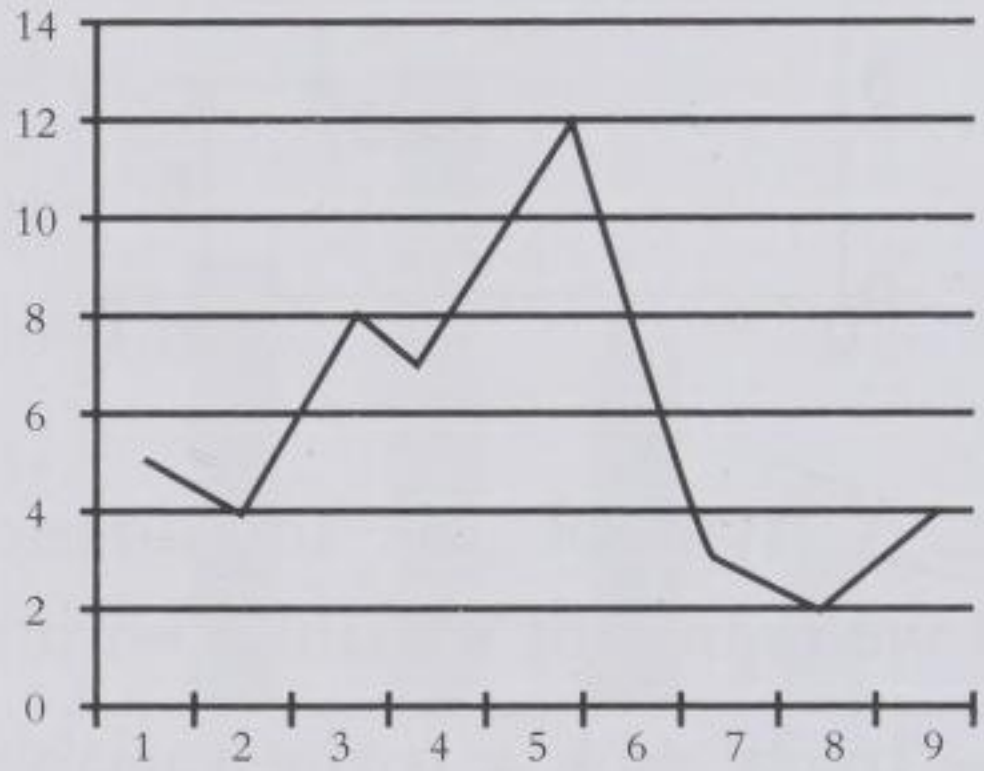
$$= \frac{15 + 13 + 18 + 16 + 14 + 17 + 12}{7}$$

$$= \frac{105}{7}$$

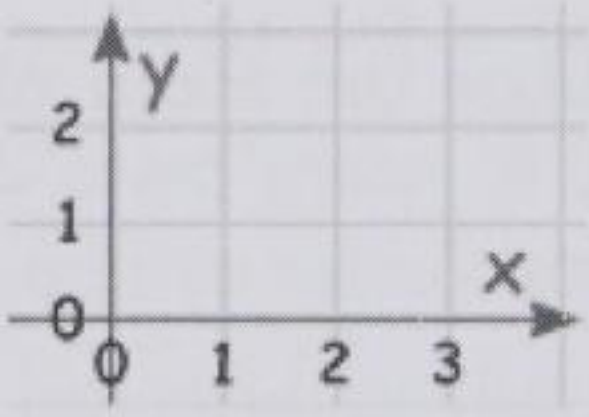
$$= 15$$

10. **Line graph**: Line graphs compare two variables.

Each variable is plotted along an axis. A line graph has a vertical axis and a horizontal axis.



11. **Axes**: It is the plural form of axis. The axes generally mean two lines, horizontal and vertical, crossing each other at 90 degree. The horizontal line is called x-axis and the vertical line is called y-axis.



12. **Scale model**: Scale model is used to draw a blueprint of roads, buildings etc. on paper. Even a GPS works on a scale model.

20

Five Nines to Ten

This is a simple problem yet a brain-teaser where you will be required to make 10 using five 9s and some mathematical operations. Can you do this? Let me explain the method with some examples where I am using four 9s to get values 0 to 9. See how I am doing it and get ready to make 10 using five 9s.

Example: Using four 9s and mathematical symbols like +, –, × and ÷, express it in digits from 0 to 9.

Solution:

$0 = 99 - 99$

$= (9/9) - (9/9)$

$1 = 99/99$

$= (9/9) \times (9/9)$

$= (9 + 9) / (9 + 9)$

$2 = (9/9) + (9/9)$

$= (99/9) - 9$

$3 = (9 + 9 + 9) / 9$

$4 = (9/\sqrt{9}) + (9/9)$

$5 = (9 - \sqrt{9}) - 9/9$

$6 = \sqrt{(9 + 9 + 9 + 9)}$

$7 = 9 - ((9 + 9)/9)$

$8 = ((9 \times 9)-9)/9$

$= (\sqrt{9} \times \sqrt{9}) - 9/9$

$9 = (9 + 9 + 9) / \sqrt{9}$

$10 = (9 + (9 \times 9))/9$

Hope you have understood the way to arrive at different values using four 9s. Let's try to make 10 in a different way using five 9s.

$10 = \sqrt{9} + \sqrt{9} + \sqrt{9} + 9 / 9$

$$10 = 9 + \frac{99}{99}$$

$$10 - 9 + 99 - \frac{9}{9}$$

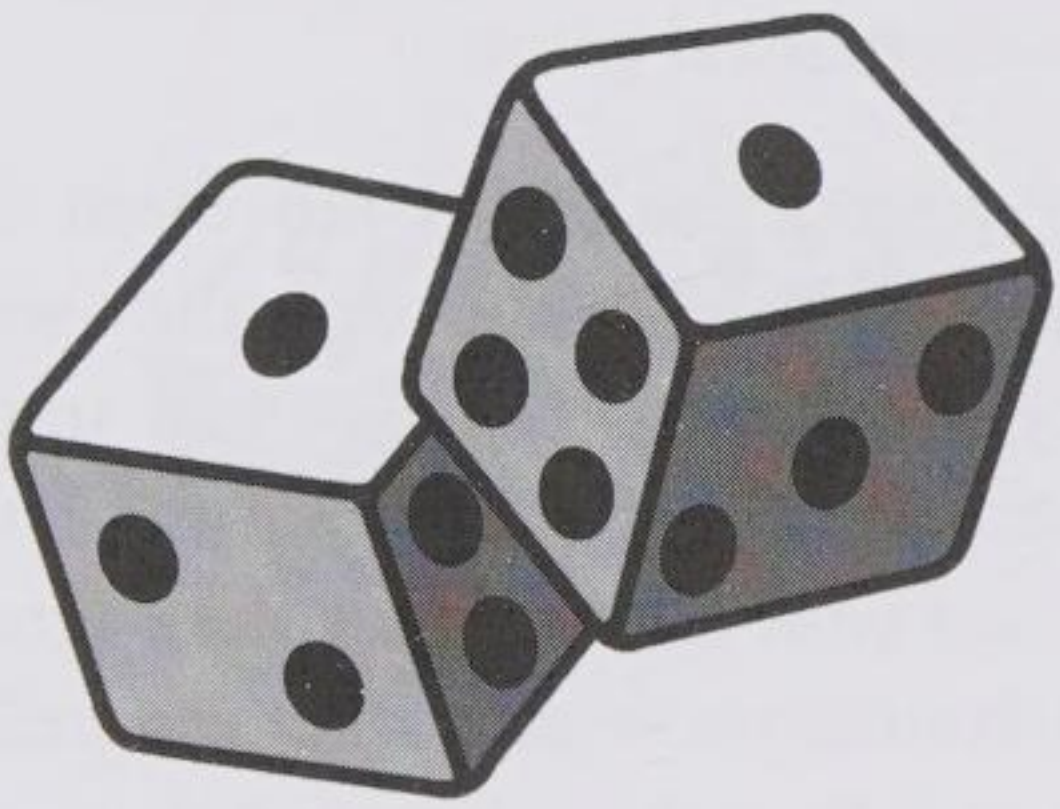
$$10 - 9 + \frac{9 - 9}{9 + 9}$$

$$10 = 9 + \frac{9 \times 9}{9 \times 9}$$

This is not all. You can find many more such brain-teasers. So what are you waiting for? Get ready to find more.

21

Game of Two Dice



Do you love playing snakes and ladders?

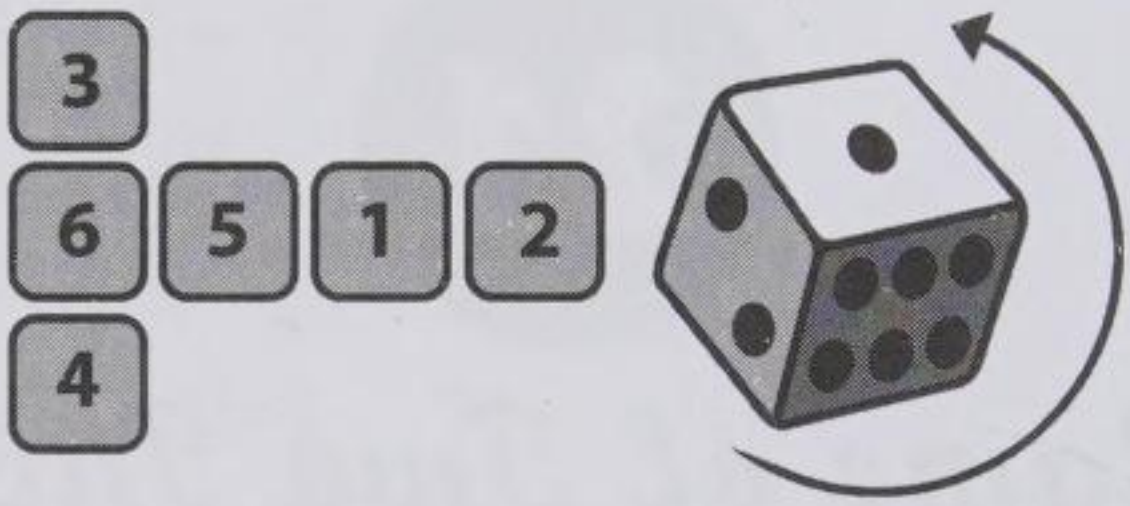
Games like ludo and snakes and ladders are played with the help of dice. A dice or a die is a cube with each of its six faces showing a different number of dots from 1 to 6.

➲ The sum of numbers on opposite sides of a dice comes to 7.

$1 + 6 = 3 + 4 = 2 + 5 = 7$

A dice is used to play many games and the player notes the outcome of the throw of a dice and moves accordingly on the playing board. The numbers on the dice are arranged,

implying that the 1, 2 and 3 faces share a vertex; these faces may be placed clockwise or counterclockwise around this vertex.



Let's play a mathematical game.

Call some of your friends and tell them that you can predict the number that appears on a dice without seeing it. They will obviously not believe you initially, but tell them that you have learnt the art of predicting, and you want them to see your talent. Once they agree to play the game, give each of them a pair of dice, a pen and a paper and tell them to follow these instructions:-

- Throw two dice and note down the numbers they throw up.
- Multiply the first number by 2.
- Add 5.
- Multiply it again by 5.
- Add the number on the second dice to the previous result.

Now tell me your final result.

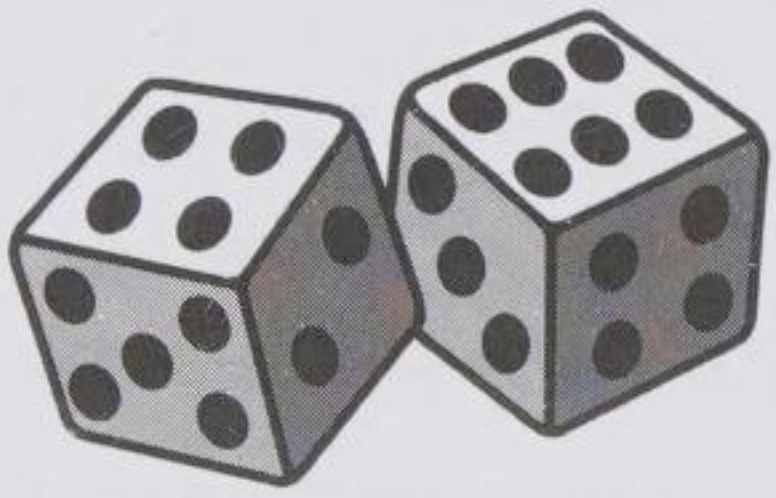
Wait for a minute and let me do my magic!

I can now tell you what the two numbers on the dice are.

Trick: Subtract 25 from the last result and you will obtain a two-digit number. The digit at the ten's place will tell you the number of the first dice and the digit at unit's place will tell you the number of the second dice.

Let's understand this puzzle with an example.

Example: Suppose you throw a pair of dice and the numbers appearing on them are 4 and 6 respectively.



Here, first number = 4 and second number = 6

Let's check the puzzle by following the steps discussed earlier.

- Multiply the first number by 2 ⇨ 4 ×2 = 8.
- Add 5 ⇨ 8 + 5 = 13.
- Multiply it again by 5 ⇨ 13 × 5 = 65.
- Add the number on the second dice to the previous result ⇨ 65 + 6 = 71.

Final result = 71

Subtract 25 from it ⇨ 71 – 25 = 46

As you can see, the number at ten's place = 4 = First number

Number at one's place = 6 = Second number

Is it magic or mathematics?

It is obviously not any magic, but math-a-magic.

Let's understand the whole process with the help of algebra.

First number on the dice = x

Second number on the dice = y

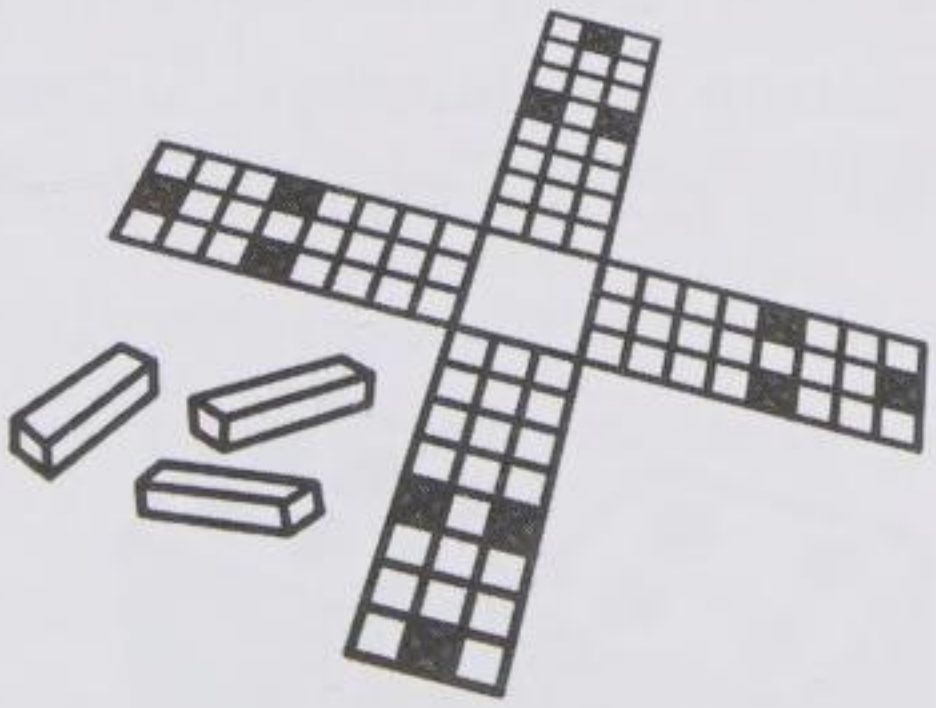
- Multiply the first number by 2 ⇨ $2x$
- Add 5 ⇨ $2x + 5$
- Multiply it again by 5 ⇨ $10x + 25$
- Add the number that appeared on the second dice to the previous result ⇨ $10x + y + 25$

If you subtract 25 from it, you get $10x + y$, which is the expression for writing the two-digit number.

22

Game of Three Dice

Indians were big on gambling around 3000 BC and the use of dice was quiet common during those days. You must have heard one game known as *Pachisi* played by 2 to 4 players in which a player getting 25 first wins the game.



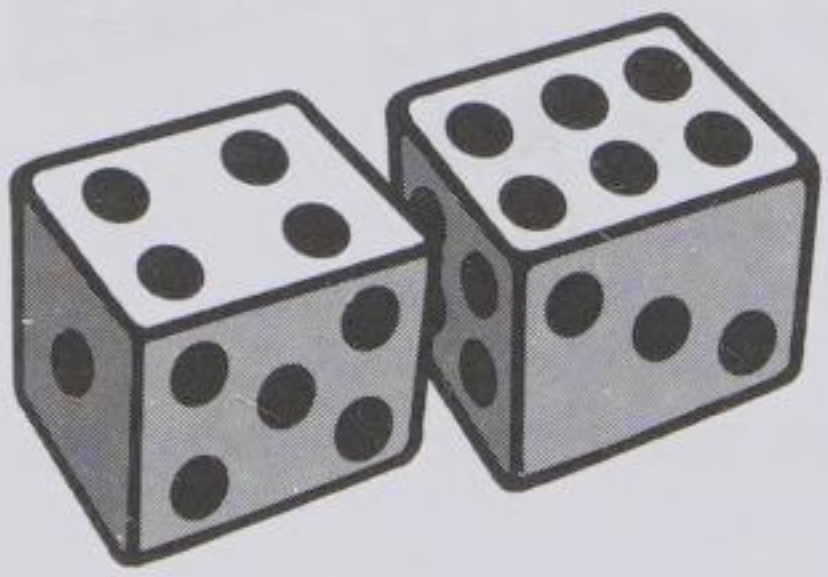
During those days, the shape of dice was not cubical, but rectangular.

Let's play a mathematical game with the help of three dice and enjoy the beauty of maths. Call some of your

friends, give them three dice each and tell them that you can predict the number that will appear on them. This game will require some mathematical calculation so you can provide calculators to your friends and ask them to follow these steps. Get ready for some fun!

- Throw three dice and note down the number that appears on them.
- Multiply the first number by 2.
- Add 5 to it.
- Multiply the result by 5.
- Add the second number to the previous result.
- Multiply the result by 10.
- Add the third number to the previous result.
- Subtract 125 from the result and get the final number..

Once you get the final result, take some time and pretend to be busy calculating the number. Subtract 125 again from the last result to get the numbers in the dice in the correct order.



Let's take an example.

Example: Suppose the numbers on the three dice show up as 3, 2 and 6

1st number = 3; 2nd number = 2; 3rd number = 6

- Multiply the 1st number by 2 ⇨ $2 \times 3 = 6$.
- Add 5 to it ⇨ $6 + 5 = 11$.
- Multiply the result by 5 ⇨ $11 \times 5 = 55$.
- Add the number that appeared on the 2nd dice to the previous result ⇨ $55 + 2 = 57$.
- Multiply the result by 10 ⇨ $57 \times 10 = 570$.
- Add the 3rd number to the previous result ⇨ $570 + 6 = 576$.
- Subtract 125 from the result and get the final result ⇨ $576 - 125 = 451$.
- Final result = 451.

Subtract 125 from the final result ⇨ $451 - 125 = 326$

Number at hundred's place = 3 = 1st number

Number at ten's place = 2 = 2nd number

Number at one's place = 6 = 3rd number

Do you know that you are using the principle of arithmetic and algebra while playing this beautiful game?

Let's first write 326 in standard form.

$326 = 3 \times 100 + 2 \times 10 + 6$

If x is the digit in the hundred's place, y is the digit in the ten's place and z is the digit in one's place, then the number = $100x + 10y + z$

- Multiply the 1st number by 2 ⇨ $2x$.
- Add 5 to it ⇨ $2x + 5$.

- Multiply the result by 5 ⇨$10x + 25$.
- Add the second number to the previous result ⇨ $10x + 25 + y$
- Multiply the result by 10 ⇨ $100x + 10y + 250$
- Add the 3rd number to the previous result ⇨ $100x + 10y + 250 + z$
- Subtract 125 from the result and tell me the final number ⇨ $100x + 10y + 250 + z - 125$
- Final result ⇨ $100x + 10y + 125 + z$

Finally, subtract 125 from the last result ⇨ $100x + 10y + z + 125 - 125 = 100x + 10y + z$.

Dice Constant 49

As you know, the sum of the numbers on the top and the bottom of a dice is 7. In this puzzle, I shall explore this property of dice and play the game with two dice. Call some of your friends and tell them to play a game with two dice. The best part of this game is that that you know the result well in advance. The result is 49.

So take a paper and write 49 on it and put it in your pocket. Give a pair of dice, a pen and a piece of paper to a friend with whom you will play this game. Ask him/her to follow these instructions:

- Throw a pair of dice.
- Write the numbers that appear on top of both dice.
- Ask your friend to write down the numbers that appear on the reverse side of the dice.
- Multiply the top digits on both dice.
- Multiply the bottom digits on both dice.
- Multiply the top digit of the 1st dice with the bottom digit of the 2nd dice.

- Multiply the top digit of the 2nd dice with the bottom digit of the 1st dice.

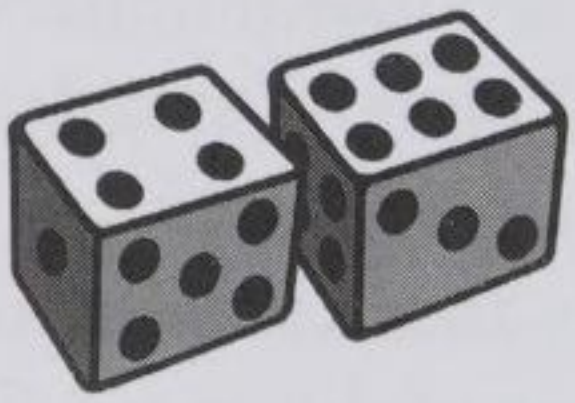
Add the numbers.

Is the sum 49?

Oh my god!

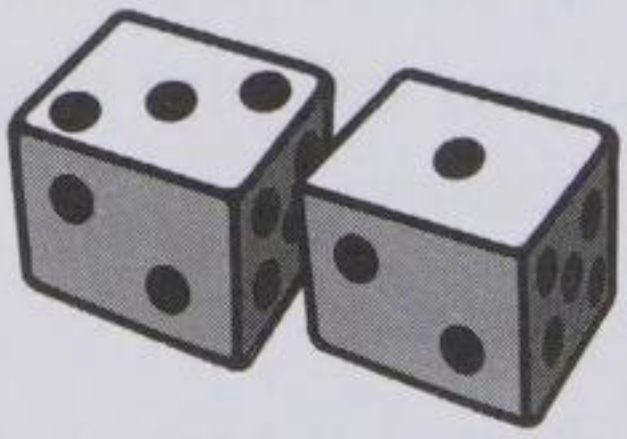
You guessed the result correctly.

Before explaining the reason behind it, let us take an example and try to check the validity of the result. Let the numbers on two dice thrown be:-



Numbers at the top are 4 and 6.

The number below 4 will be 3 and that below 6 will be 1.



The numbers at the bottom are 3 and 1.

Therefore,

- Multiply the top digits on both dice ⇨ $4 \times 6 = 24$
- Multiply the bottom digits on both dice ⇨ $3 \times 1 = 3$

- Multiply the top digit of the 1st dice with the bottom digit of the 2nd dice ⇨ $4 \times 1 = 4$
- Multiply the top digit of the 2nd dice with the bottom digit of the 1st dice ⇨ $6 \times 3 = 18$

Sum of the four ⇨ $24 + 3 + 4 + 18 = 49$

Now it is your turn to show your magic. Take out the slip with 49 written on it and show it to your friend and tell him/her that you knew the sum in advance!

Remember that the sum of the four entries will always be 49, whatever the number at the top is.

Let's understand it mathematically.

Suppose the two numbers on the top are x and y.

Number at the bottom = $7 - x$ and $7 - y$

- Multiply the top digits ⇨ $x \times y = xy$.
- Multiply the bottom digits ⇨ $(7 - x) \times (7 - y) = 49 - 7x - 7y + xy$
- Multiply the top digit of the 1st dice with the bottom digit of the 2nd dice ⇨ $x\,(7 - y)$.
- Multiply the top digit of the 2nd dice with the bottom digit of the 1st dice ⇨ $y\,(7 - x)$

Sum = $xy + 49 - 7x - 7y + xy + 7x - xy + 7y = 49$

24

Birthday Puzzle

How would you react if someone claims that they can predict your birthday?

You will obviously call them a fortune teller; but if you know a little bit of mathematics and love doing some simple mathematical operations, then you too can be that fortune teller and tell the birthday of any of your friends or relatives.

Let's get started:

Take your pen, paper and a calculator. If you are using a mobile phone then you can directly calculate on your phone. You simply have to follow the instructions given below to see the magic of mathematics.

➲ Add 18 to the month in which you were born. (January = 1, February = 2, March = 3, April = 4, May = 5, June = 6, July = 7, August = 8, September = 9, October = 10, November = 11 and December = 12)

- Multiply the result by 25.
- Subtract 333 from the previous result.
- Multiply it by 8.

Feeling tired? Rest for a minute or two!

Now, follow the remaining steps:

- Subtract 554 from your previous result.
- Divide it by 2.
- Add your birth day.
- Multiply the result by 5.
- Add 692 to it.
- Multiply it by 20.
- Add the last 2 digits of the year in which you were born. (74 for 1974, 83 for 1983 and 14 for 2014)

Now your wait is over. There is only one more step to go.

- Subtract 32940 from your previous result and give me the final result.

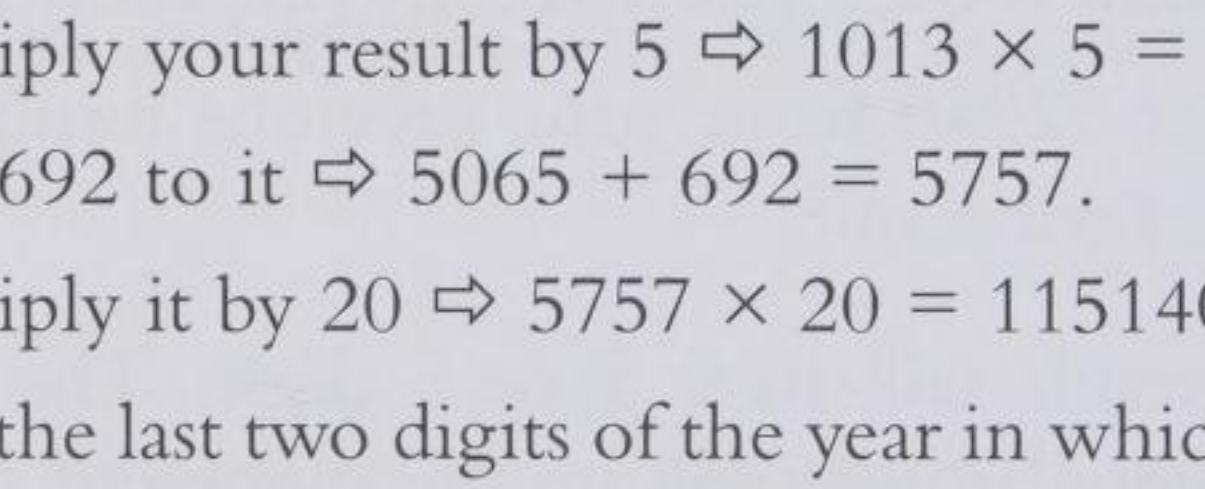
Your final result will have 5 or 6 digits. The first 1 or 2 digits of your answer will be the month of your birth, the middle 2 digits will tell you the date you were born and the remaining 2 digits will give you the year you were born in. Calculate from the right—that is, taking the last 2 digits (birth year) first. If the answer is a 5-digit number, then taking the first 2 digits will provide an incorrect answer.

Let's take an example.

Example:- Suppose you are playing this game with Mr X who was born on August 22, 1964 (22/08/1964), then he will do the following calculations in his notebook.

- Add 18 to the month in which you were born ⇨ 8 + 18 = 26.
- Multiply the result by 25 ⇨ 26 × 25 = 650.
- Subtract 333 from the previous result ⇨ 650 – 333 = 317.
- Multiply it by 8 ⇨ 317 × 8 = 2536.
- Subtract 554 from your previous result ⇨ 536 – 554 = 1982.
- Divide it by 2 ⇨ 1982 ÷ 2 = 991.
- Add your birth day ⇨ 991 + 22 = 1013.
- Multiply your result by 5 ⇨ 1013 × 5 = 5065.
- Add 692 to it ⇨ 5065 + 692 = 5757.
- Multiply it by 20 ⇨ 5757 × 20 = 115140.
- Add the last two digits of the year in which you were born ⇨ 115140 + 64 = 115204.
- Subtract 32940 from your previous result and give me the final result

⇨ 115204 – 32940 = 82264



Isn't it magical?

Yes, indeed it is a magic of mathematics.

Let's find the secret with the help of algebra.

We generally write our birthday in the following format.

D	D	M	M	Y	Y

For the sake of convenience, we'll call DD = x, MM = y and YY = z.

Let's do the calculation.

- Add 18 to the month in which you were born ⇨ $18 + y$.
- Multiply the result by 25 ⇨ $25y + 450$.
- Subtract 333 from the previous result ⇨ $25y + 450 - 333 = 25y + 117$.
- Multiply it by 8 ⇨ $200y + 936$.
- Subtract 554 from your previous result ⇨ $200y + 936 - 554 = 200y + 382$.
- Divide it by 2 ⇨ $(200y + 382) \div 2 = 100y + 191$.
- Add your birthday ⇨ $100y + 191 + x$.
- Multiply your result by 5 ⇨ $(100y + 191 + x\ ) 5 = 500y + 955 + 5x$.
- Add 692 to it ⇨ $500y + 955 + 5x + 692 = 500y + 5x + 1647$.
- Multiply it by 20 ⇨ $(500y + 5x + 1647) \times 20 = 10000y + 100x + 32940$.

- Add the last two digits of the year in which you were born ⇨ $10000y + 100x + 32940 + z$.
- Subtract 32940 from your previous result ⇨ $10000y + 100x + 32940 + z - 32940 = 10000y + 100x + z$.

Now check the expression by putting the value of

$y = 08; x = 22; z = 64$

We get $80000 + 2200 + 64 = 82264$, which proves our result.

25

Ramanujan Birthday Square

Have you heard about the Indian prodigy, mathematician Srinivas Ramanujan?

Srinivas Iyengar Ramanujan was born in a Brahmin family on December 22, 1887 in the city of Erode in Madras Presidency. His father, K Srinivasa Iyengar, worked as a clerk in a sari shop and hailed from the district of Thanjavur. His mother, Komalatammal, was a housewife and also sang at a local temple. They lived in Sarangapani

Street in a traditional home in the town of Kumbakonam. The family home is now a museum. Ramanujan has done extensive research in Mathematics and contributed to number theory, hyper geometric series, partition number and many more theories under the able guidance of Professor G H Hardy. His birthday, December 22, is celebrated as National Mathematics Day in India. So let's salute this genius with a 4 x 4 magic square that is based on his birth date, i.e. 22/12/1887. This birthday magic square was also constructed by Ramanujan.

22	12	18	87
88	17	9	25
10	24	89	16
19	86	23	11

As you can see, the first row of this magic square is the date of birth of Srinivasa Ramanujan. Before I discuss the rule of constructing a magic square of your birthday, let me tell you some important properties of this magic square.

Properties of Ramanujan Birthday Square

- Like any other magic square, the sum of each row, column and its diagonal is 139.

22 + 12 + 18 + 87 = 139
88 + 17 + 9 + 25 = 139
10 + 24 + 89 + 16 = 139
19 + 86 + 23 + 11 = 139

Sum of Rows

22 + 88 + 10 + 19 = 139
12 + 17 + 24 + 86 = 139
18 + 9 + 89 + 23 = 139
87 + 25 + 16 + 11 = 139

Sum of Columns

22 + 17 + 89 + 11 = 139
19 + 24 + 9 + 87 = 139

Sum of Diagonals

- This square has one additional property. Sum of all the corner numbers of this square is also 139.

  22 + 87 + 19 + 11 = 139

  88 + 25 + 10 + 16 = 139 (sum of corner numbers of two horizontal middle rows).

- The sum of the middle square is also 139.

17	9
24	89

Let's learn how to construct the birthday magic square. Looking upon the birthday square of Ramanujan you can assign the algebraic value to this magic square, as shown on the next page.

A	B	C	D
D+1	C-1	B-3	A+3
B-2	A+2	D+2	C-2
C+1	D-1	A+1	B-1

Now you can construct the birthday magic square for any friend with the help of the table given above.

If I was born on March 24, 1975, then my birthday magic square will look like

A = 24	B = 03	C = 19	D = 75
D + 1 = 76	C – 1 = 18	B – 3 = 0	A + 3 = 27
B – 2 = 1	A + 2 = 26	D + 2 = 77	C – 2 = 17
C + 1 = 20	D – 1 = 74	A + 1 = 25	B – 1 = 2

You can easily see that the magic constant of my birthday magic square is 121.

So, the next time when you get an invitation from a friend on their birthday, never forget to present a birthday magic square as a gift.

Books for Further Reading

1. *Challenging Math Puzzles* – Glen Vecchione
2. *Amazing Math Puzzles* – Adam Hart-Davis
3. *The Moscow Puzzles: 359 Mathematical Recreations* – Boris Kordemsky
4. *Wonders of Numbers: Adventures in Mathematics, Mind, and Meaning* – Clifford A Pickover
5. *Recreations in Mathematics* – HE Licks
6. *Math Puzzles and Games, Grades 6-8: Over 300 Reproducible Puzzles that Teach Math and Problem-Solving Skills* – Terry Stickels
7. *Entertaining Mathematical Puzzles* – Martin Gardner
8. *Brain Sharpeners* – AR Rao

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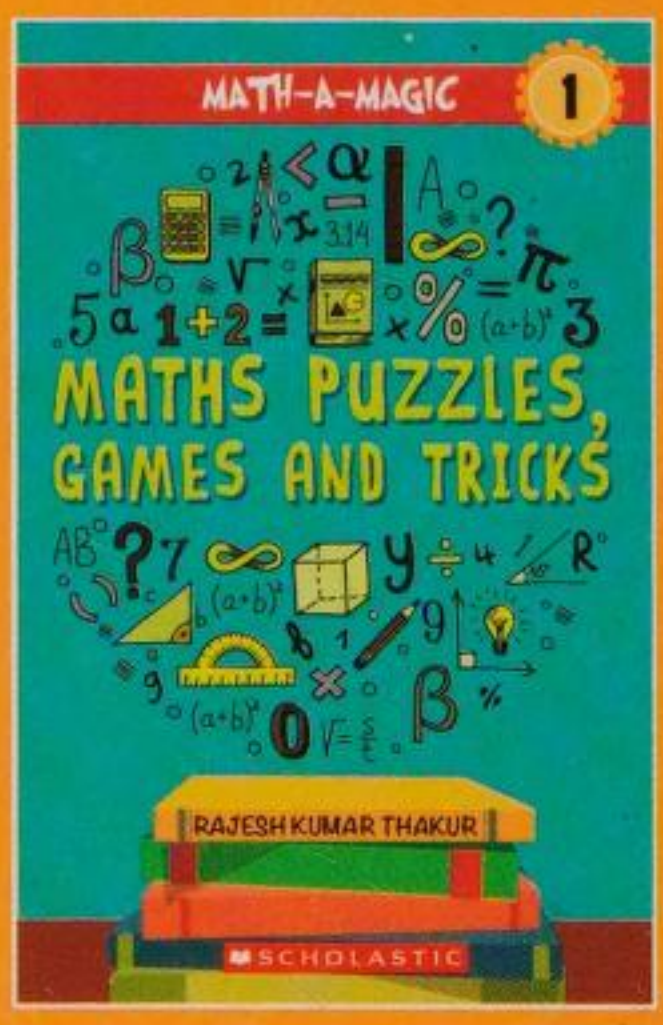

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